

First-Order Logic

Part One

Recap from Last Time

Recap So Far

- A ***propositional variable*** is a variable that is either true or false.
- The ***propositional connectives*** are as follows:
 - Negation (“not”): $\neg p$
 - Conjunction (“and”): $p \wedge q$
 - Disjunction (“or”): $p \vee q$
 - Truth (“true”): \top
 - Falsity (“false”): \perp
 - Implication (“implies”): $p \rightarrow q$
 - Biconditional (“if and only if”): $p \leftrightarrow q$

p	q	$p \rightarrow q$
F	F	—
F	T	—
T	F	—
T	T	—

What's the truth table for the \rightarrow connective?

Go to
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What's the negation of $p \rightarrow q$?

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Operator Precedence

- How do we parse this statement?

$$\neg x \rightarrow y \vee z \rightarrow x \vee y \wedge z$$

- Operator precedence for propositional logic:

\neg

\wedge

\vee

\rightarrow

\leftrightarrow

- All operators are right-associative.
- We can use parentheses to disambiguate.

Operator Precedence

- How do we parse this statement?

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- Operator precedence for propositional logic:

¬

∧

∨

→

↔

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Operator Precedence

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Operator Precedence

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$$(\neg x) \rightarrow y \vee z \rightarrow x \vee (y \wedge z)$$

- Operator precedence for propositional logic:

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Operator Precedence

- How do we parse this statement?

$$(\neg x) \rightarrow (y \vee z) \rightarrow (x \vee (y \wedge z))$$

- Operator precedence for propositional logic:

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- All operators are right-associative.
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Operator Precedence

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Operator Precedence

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$$(\neg x) \rightarrow ((y \vee z) \rightarrow (x \vee (y \wedge z)))$$

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Operator Precedence

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$$(\neg x) \rightarrow ((y \vee z) \rightarrow (x \vee (y \wedge z)))$$

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- All operators are right-associative.
- We can use parentheses to disambiguate.

Operator Precedence

- The main points to remember:
 - \neg binds to whatever immediately follows it.
 - \wedge and \vee bind more tightly than \rightarrow .
- We will commonly write expressions like $p \wedge q \rightarrow r$ without adding parentheses.
- For more complex expressions, we'll try to add parentheses.
- Confused? ***Please ask!***

Logic at the US Supreme Court

- There was a case before the US Supreme Court last year concerning whether a piece of text in a law should be interpreted as

$$\neg(A \wedge B \wedge C)$$

or as

$$\neg A \wedge \neg B \wedge \neg C.$$

- You can [listen to the oral arguments](#) on the Supreme Court's website. They regularly throw around terms like “conjunction,” “disjunction,” and “distributive.”

New Stuff!

First-Order Logic

What is First-Order Logic?

- ***First-order logic*** is a logical system for reasoning about properties of objects.
- Augments the logical connectives from propositional logic with
 - ***predicates*** that describe properties of objects,
 - ***functions*** that map objects to one another, and
 - ***quantifiers*** that allow us to reason about multiple objects.

Some Examples

$$\text{Likes}(\text{You}, \text{Eggs}) \wedge \text{Likes}(\text{You}, \text{Tomato}) \rightarrow \text{Likes}(\text{You}, \text{Shakshuka})$$


Likes(You, Eggs) \wedge Likes(You, Tomato) \rightarrow Likes(You, Shakshuka)

Learns(You, History) \vee ForeverRepeats(You, History)

In(MyHeart, Havana) \wedge TookBackTo(Him, Me, EastAtlanta)

Likes(You, Eggs) \wedge Likes(You, Tomato) \rightarrow Likes(You, Shakshuka)

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These blue terms are called *constant symbols*. Unlike propositional variables, they refer to *objects*, not *propositions*.

Likes(You, Eggs) \wedge Likes(You, Tomato) \rightarrow Likes(You, Shakshuka)

Learns(You, History) \vee ForeverRepeats(You, History)

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The red things that look like function calls are called **predicates**. Predicates take objects as arguments and evaluate to true or false.

Likes(You, Eggs) \wedge Likes(You, Tomato) \rightarrow Likes(You, Shakshuka)

Learns(You, History) \vee ForeverRepeats(You, History)

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Likes(You, Eggs) \wedge Likes(You, Tomato) \rightarrow Likes(You, Shakshuka)

Learns(You, History) \vee ForeverRepeats(You, History)

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What remains are traditional propositional connectives. Because each predicate evaluates to true or false, we can connect the truth values of predicates using normal propositional connectives.

Reasoning about Objects

- To reason about objects, first-order logic uses ***predicates***.
- Examples:

Cute(Quokka)

Rivals(Stanford, Berkeley)

- Applying a predicate to arguments produces a proposition, which is either true or false.
- Typically, when you're working in FOL, you'll have a list of predicates, what they stand for, and how many arguments they take. It'll be given separately than the formulas you write.

First-Order Formulas

- Formulas in first-order logic can be constructed from predicates applied to objects:

Cute(a) → Dikdik(a) ∨ Kitty(a) ∨ Puppy(a)

Succeeds(You) ← Practices(You)

$x < 8 \rightarrow x < 137$

The less-than sign is just another predicate. Binary predicates are sometimes written in *infix notation* this way.

Numbers are not “built in” to first-order logic. They’re constant symbols just like “You” and “a” above.

Equality

- First-order logic is equipped with a special predicate $=$ that says whether two objects are equal to one another.
- Equality is a part of first-order logic, just as \rightarrow and \neg are.
- Examples:

TomMarvoloRiddle = LordVoldemort

MorningStar = EveningStar

- Equality can only be applied to **objects**; to state that two **propositions** are equal, use \leftrightarrow .

Let's see some more examples.

$$\begin{aligned} & \text{FavoriteMovieOf}(You) \neq \text{FavoriteMovieOf}(Date) \wedge \\ & \text{StarOf}(\text{FavoriteMovieOf}(You)) = \text{StarOf}(\text{FavoriteMovieOf}(Date)) \end{aligned}$$

$$\begin{aligned} & \text{FavoriteMovieOf}(You) \neq \text{FavoriteMovieOf}(Date) \wedge \\ & \text{StarOf}(\text{FavoriteMovieOf}(You)) = \text{StarOf}(\text{FavoriteMovieOf}(Date)) \end{aligned}$$

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These purple terms are **functions**. Functions take objects as input and produce objects as output.

$$\text{FavoriteMovieOf}(You) \neq \text{FavoriteMovieOf}(Date) \wedge \\ \text{StarOf}(\text{FavoriteMovieOf}(You)) = \text{StarOf}(\text{FavoriteMovieOf}(Date))$$

FavoriteMovieOf(You) ≠ FavoriteMovieOf(Date) ∧
StarOf(FavoriteMovieOf(You)) = StarOf(FavoriteMovieOf(Date))

FavoriteMovieOf(You) ≠ FavoriteMovieOf(Date) ∧
StarOf(FavoriteMovieOf(You)) = StarOf(FavoriteMovieOf(Date))

Functions

- First-order logic allows ***functions*** that return objects associated with other objects.
- Examples:

ColorOf(Money)

MedianOf(x, y, z)

$x + y$

- As with predicates, functions can take in any number of arguments, but always return a single value.
- Functions evaluate to ***objects***, not ***propositions***.

Objects and Propositions

- When working in first-order logic, be careful to keep objects (actual things) and propositions (true or false) separate.
- You cannot apply connectives to objects:
□ *Venus* → *TheSun* □
- You cannot apply functions to propositions:
□ *StarOf(IsRed(Sun) ∧ IsGreen(Mars))* □
- Ever get confused? Just ask!

The Type-Checking Table

	... operate on and produce
Connectives (\leftrightarrow , \wedge , etc.) ...	propositions	a proposition
Predicates ($=$, etc.) ...	objects	a proposition
Functions ...	objects	an object

One last (and major) change

Some muggle is intelligent.

Some muggle is intelligent.

$\exists m. (Muggle(m) \wedge Intelligent(m))$

Some muggle is intelligent.

$\exists m. (Muggle(m) \wedge Intelligent(m))$

□ is the **existential quantifier**
and says "for some choice of
m, the following is true."

The Existential Quantifier

- A statement of the form

$\exists x. \text{some-formula}$

is true if there exists a choice of x where **some-formula** is true when that x is plugged into it.

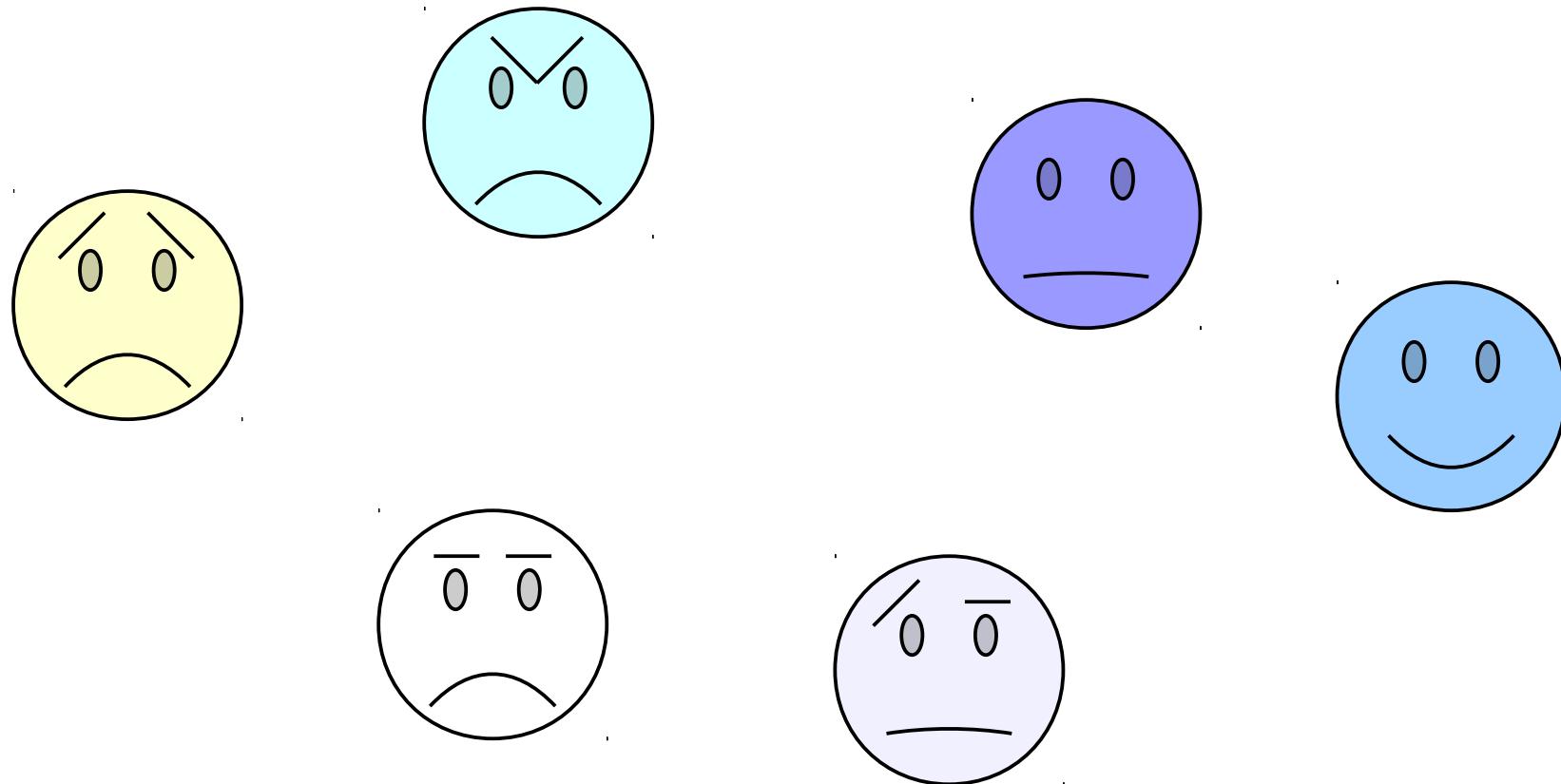
- Examples:

$\exists x. (\text{Even}(x) \wedge \text{Prime}(x))$

$\exists x. (\text{TallerThan}(x, me) \wedge \text{WeighsLessThan}(x, me))$

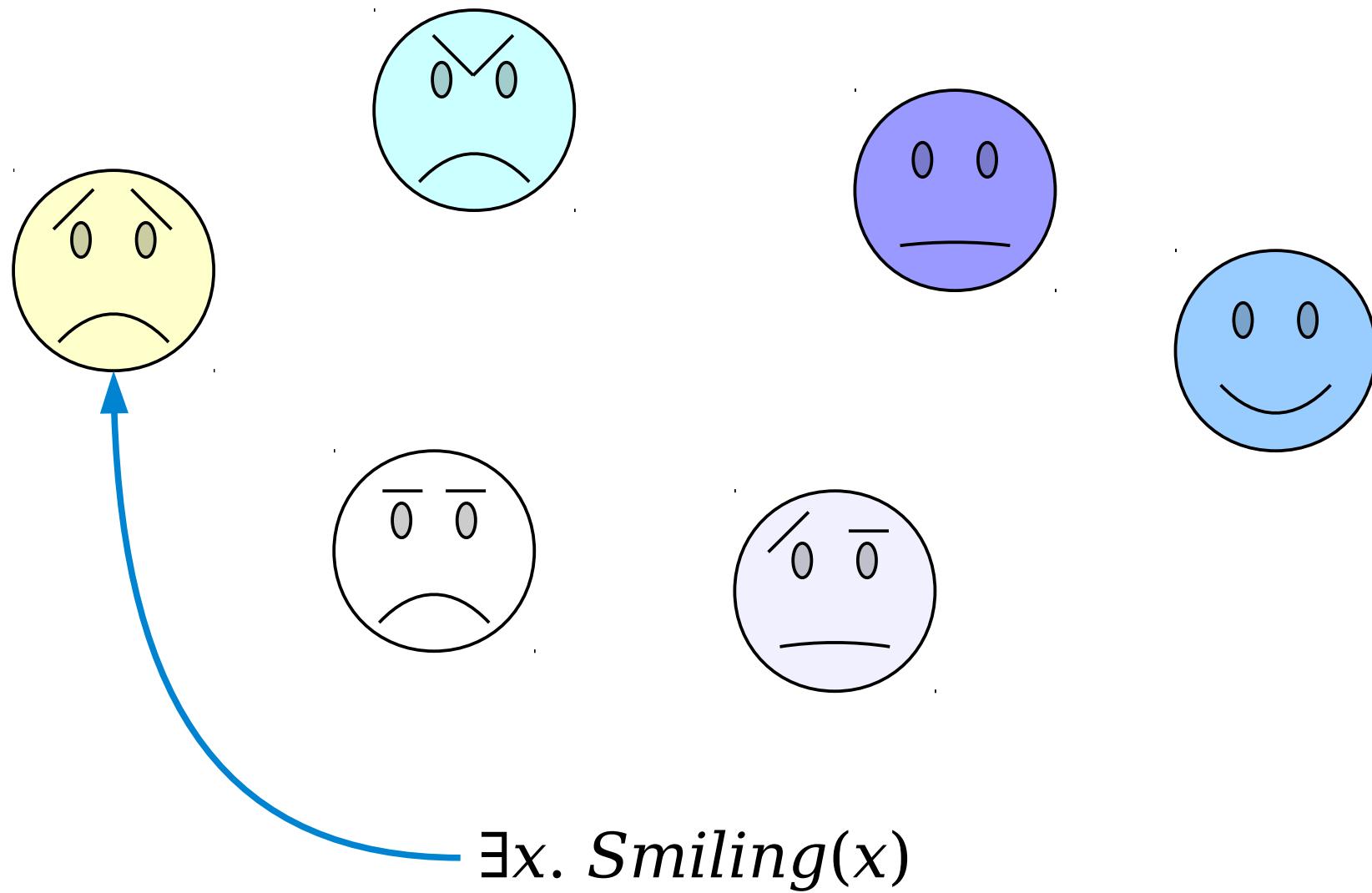
$(\exists w. \text{Will}(w)) \rightarrow (\exists x. \text{Way}(x))$

The Existential Quantifier

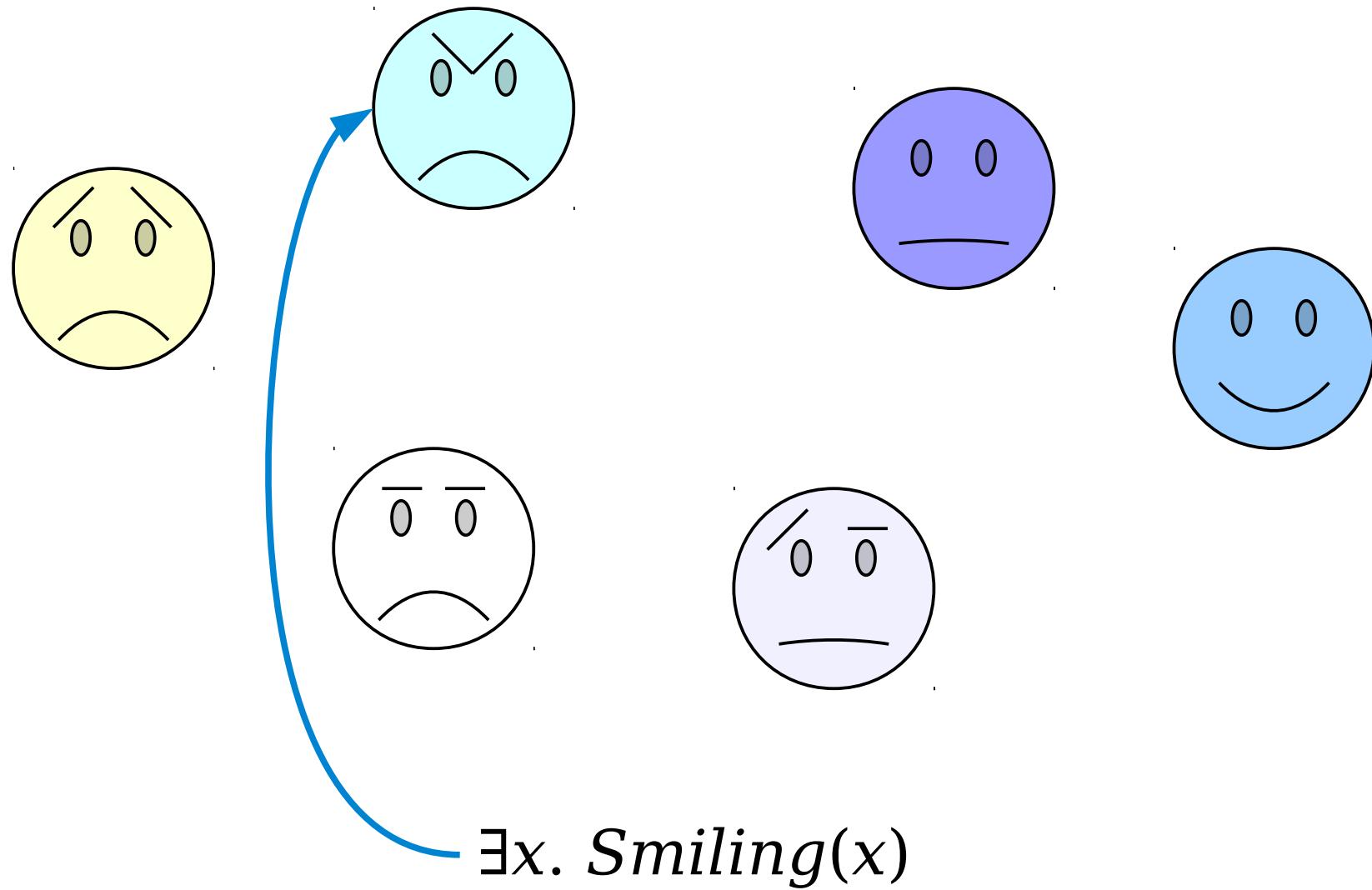


$\exists x. Smiling(x)$

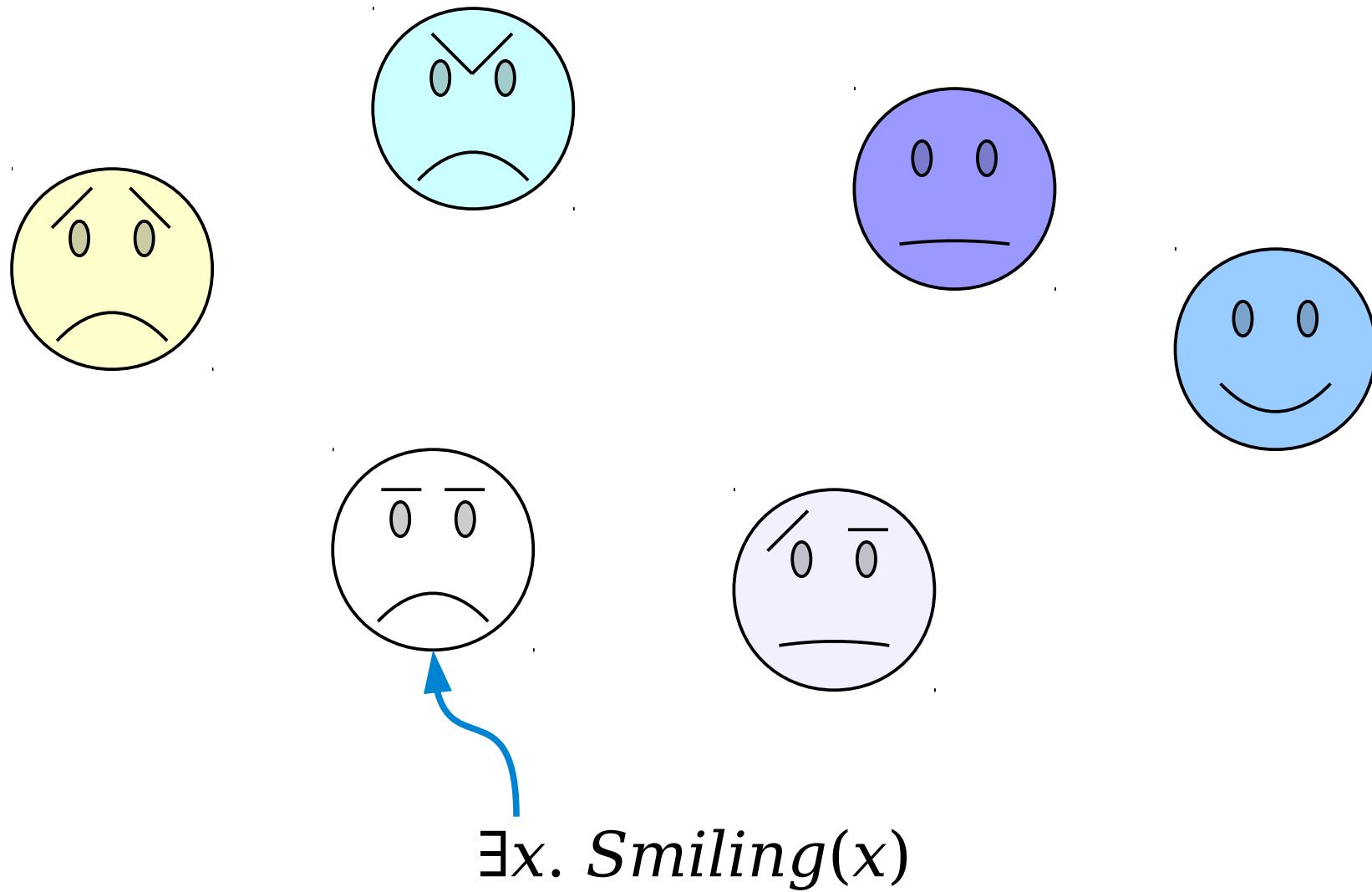
The Existential Quantifier



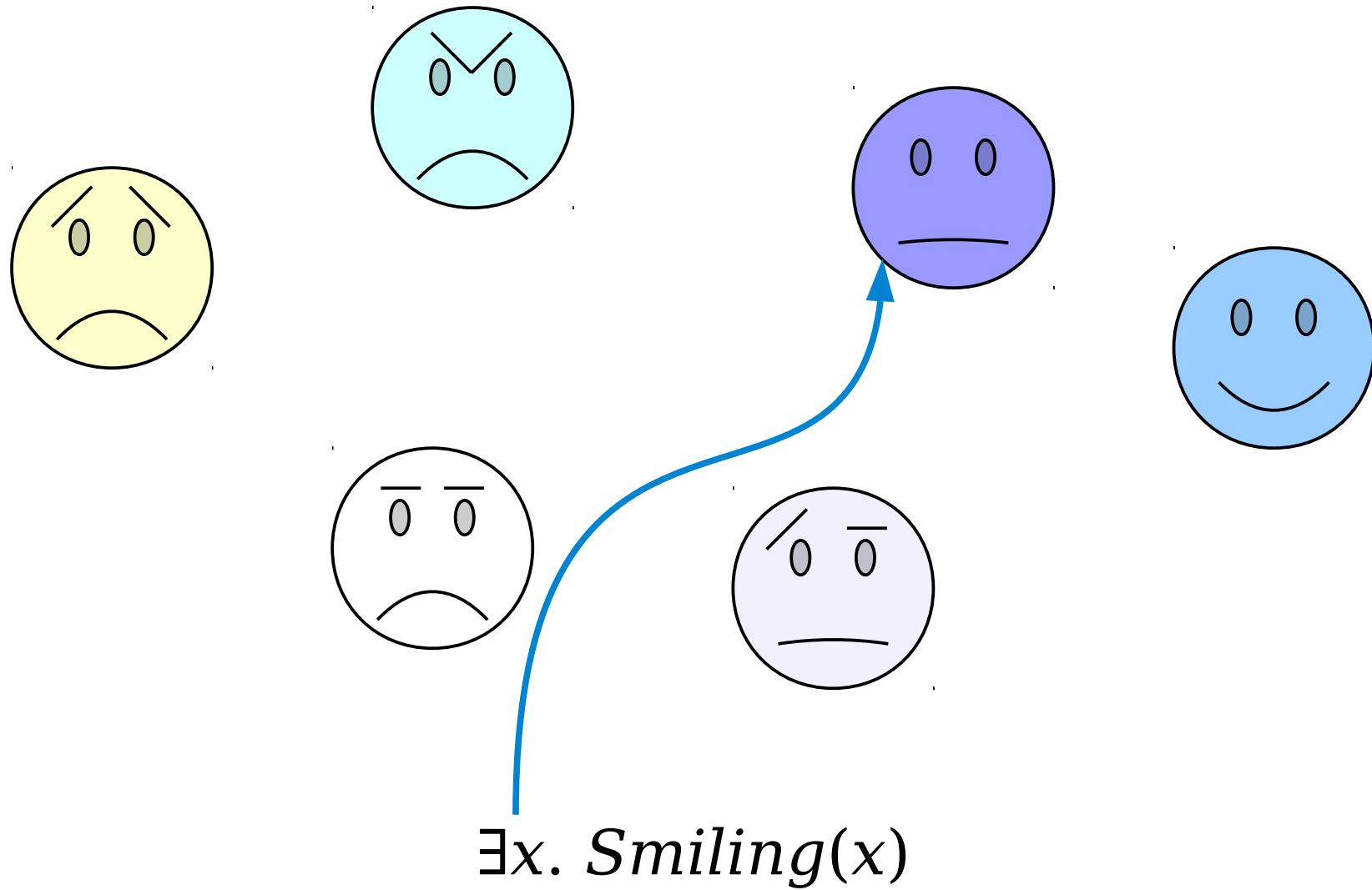
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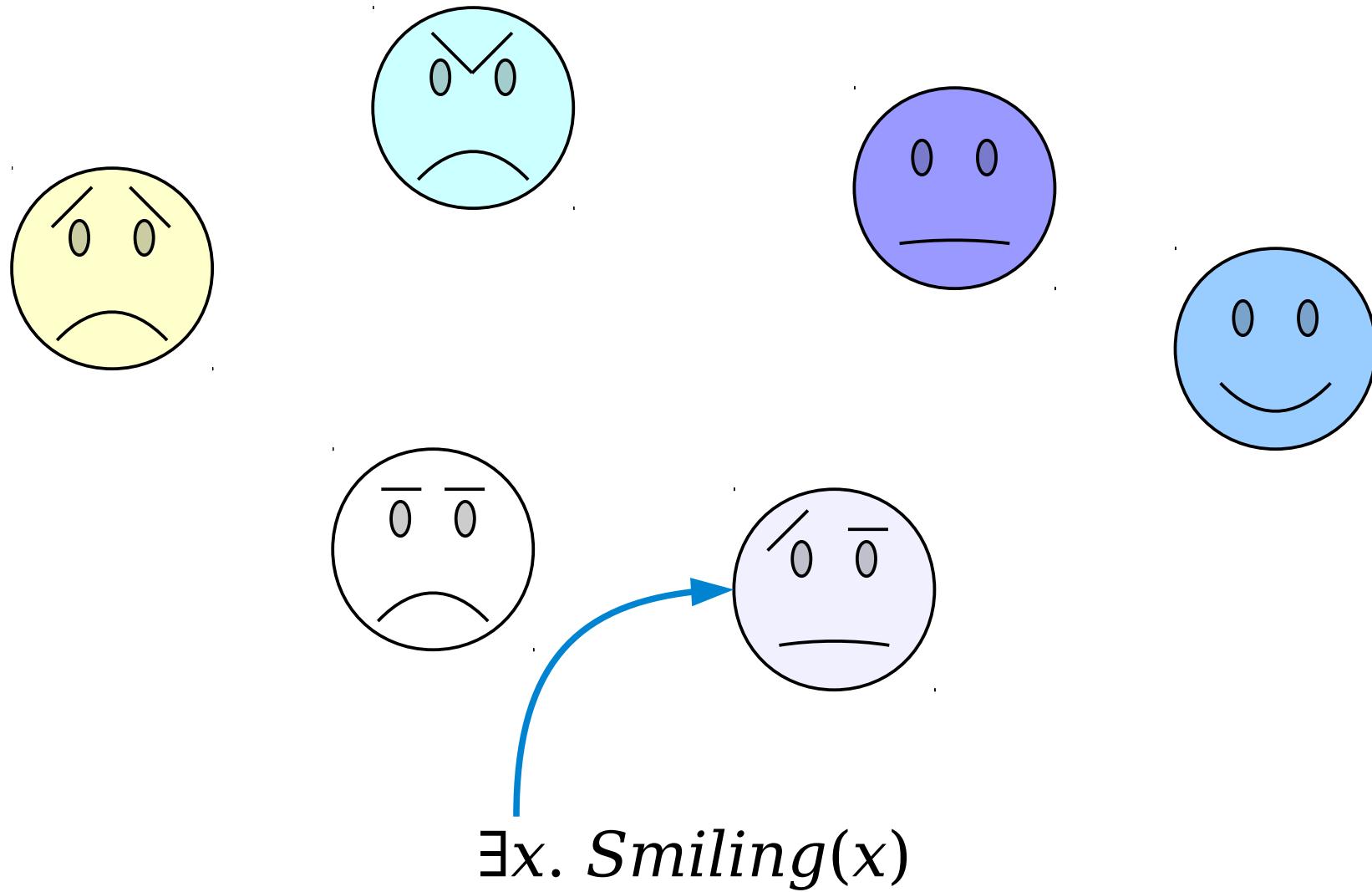
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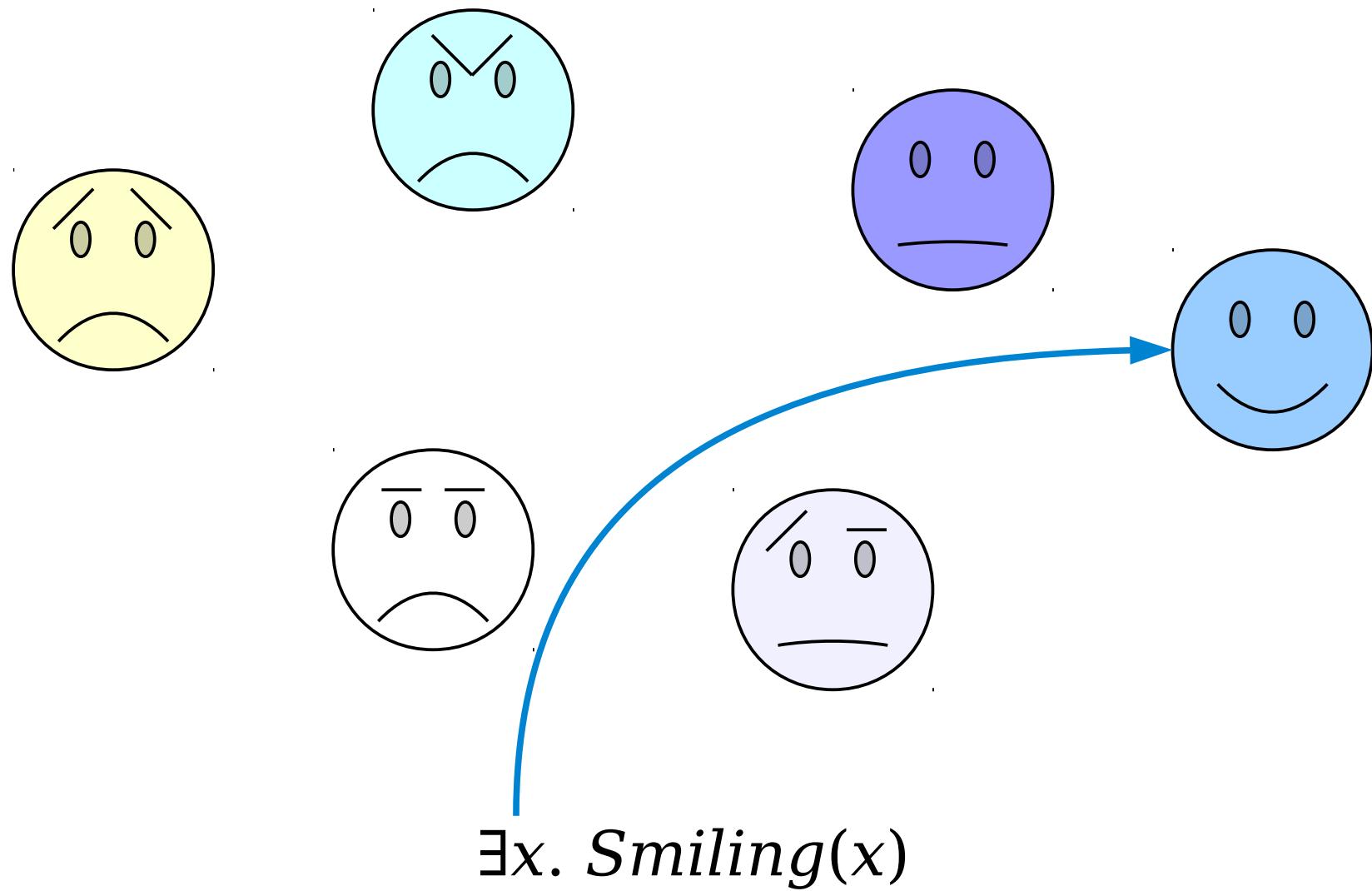
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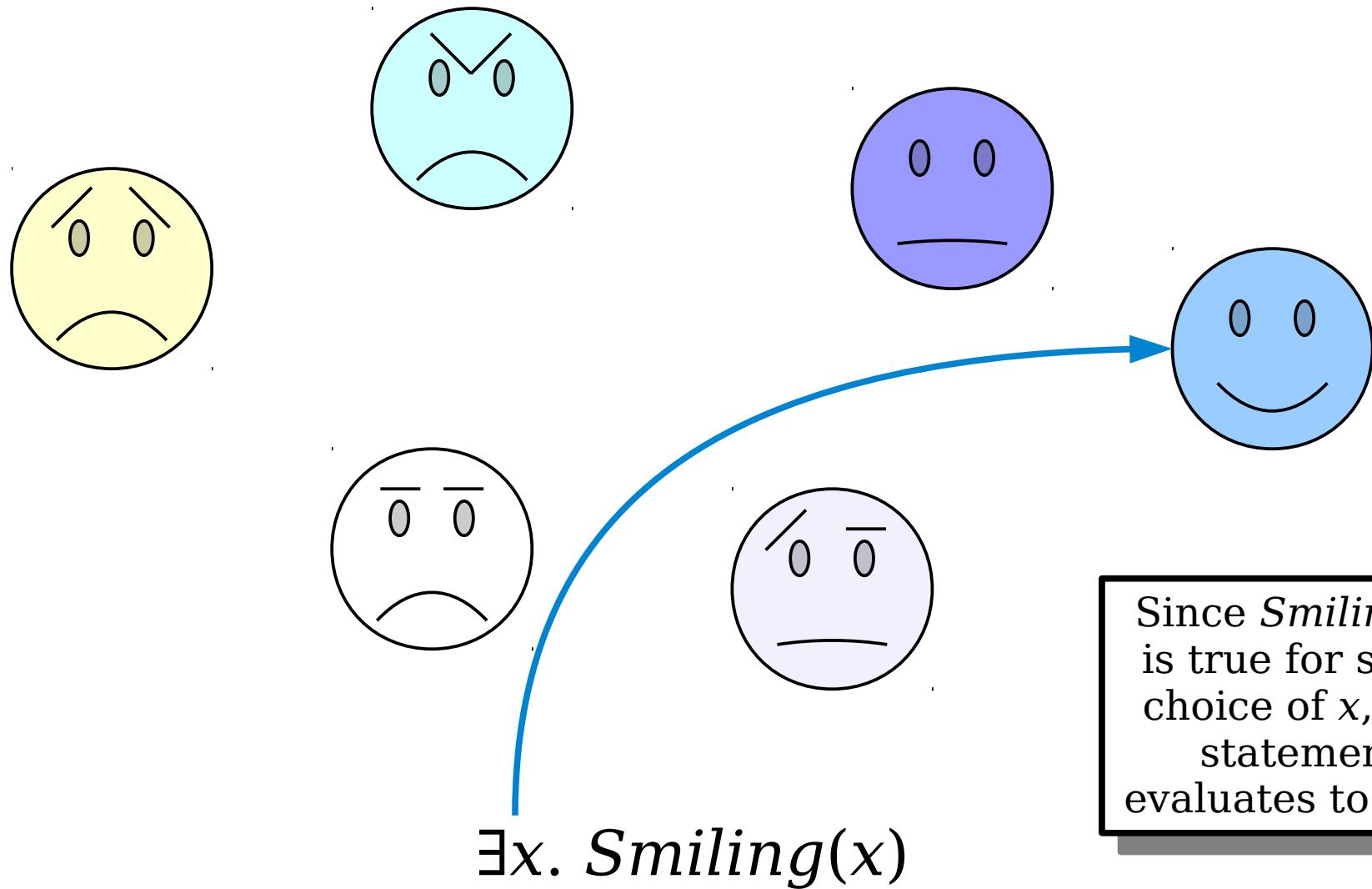
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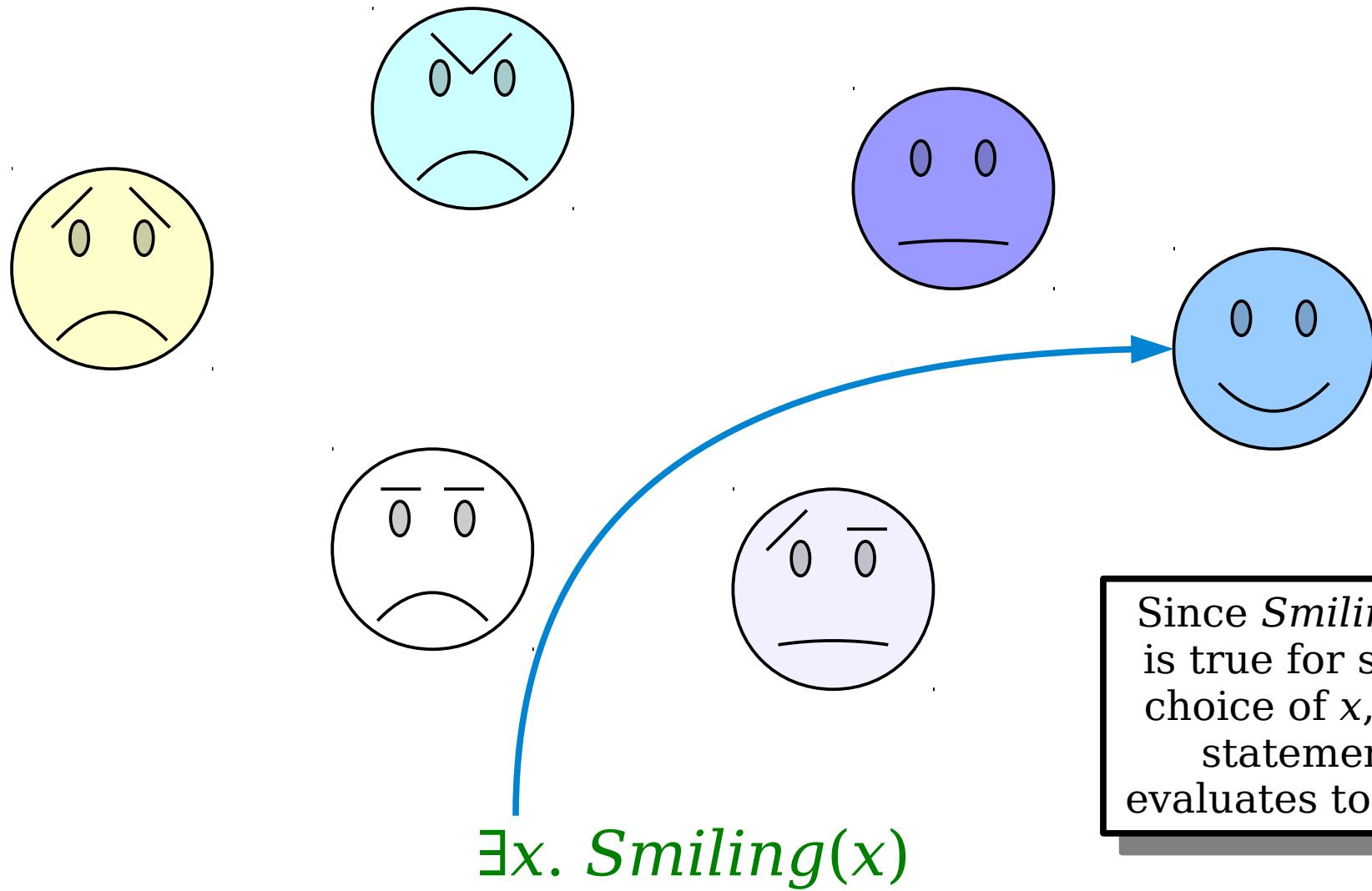
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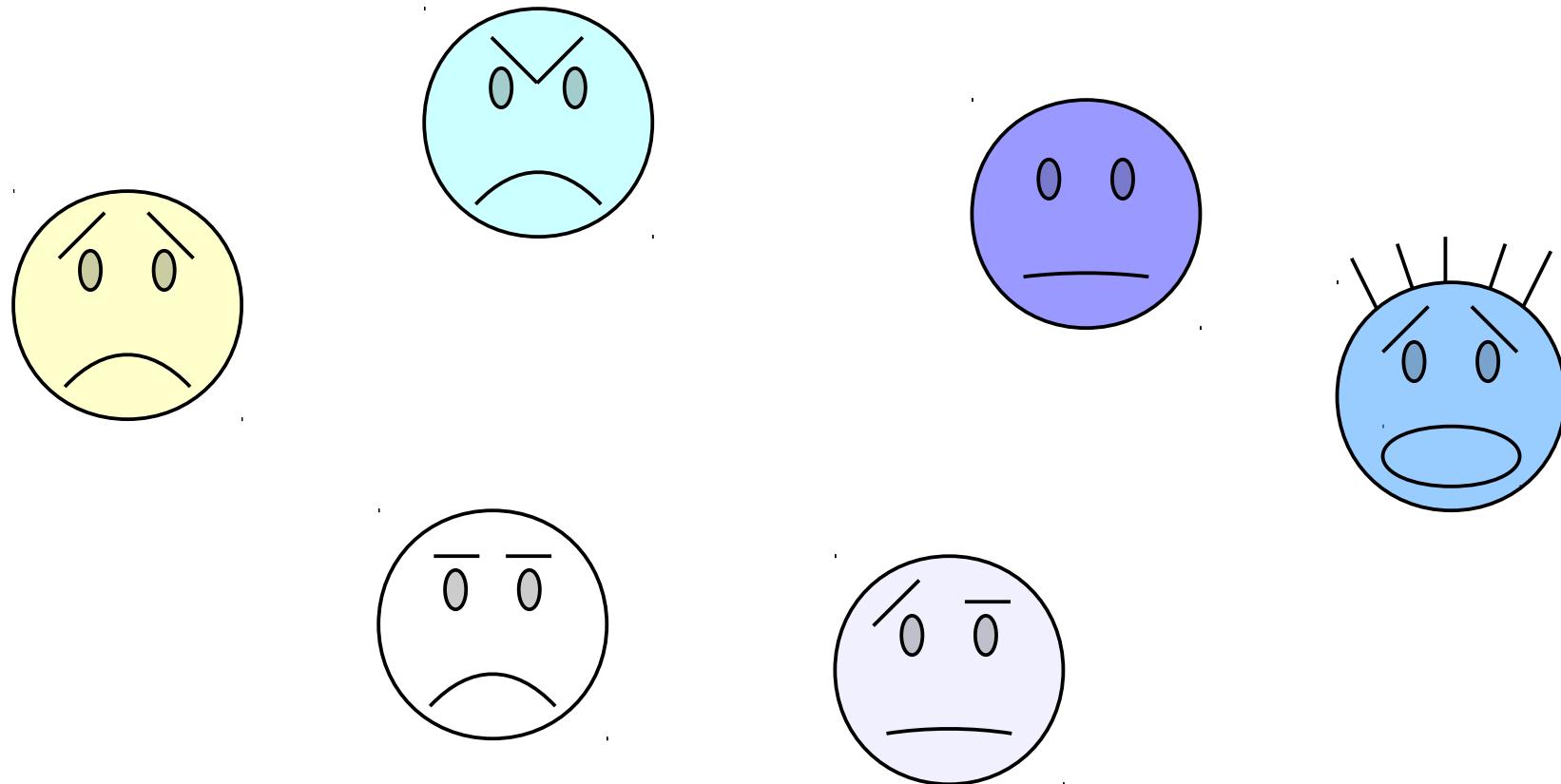
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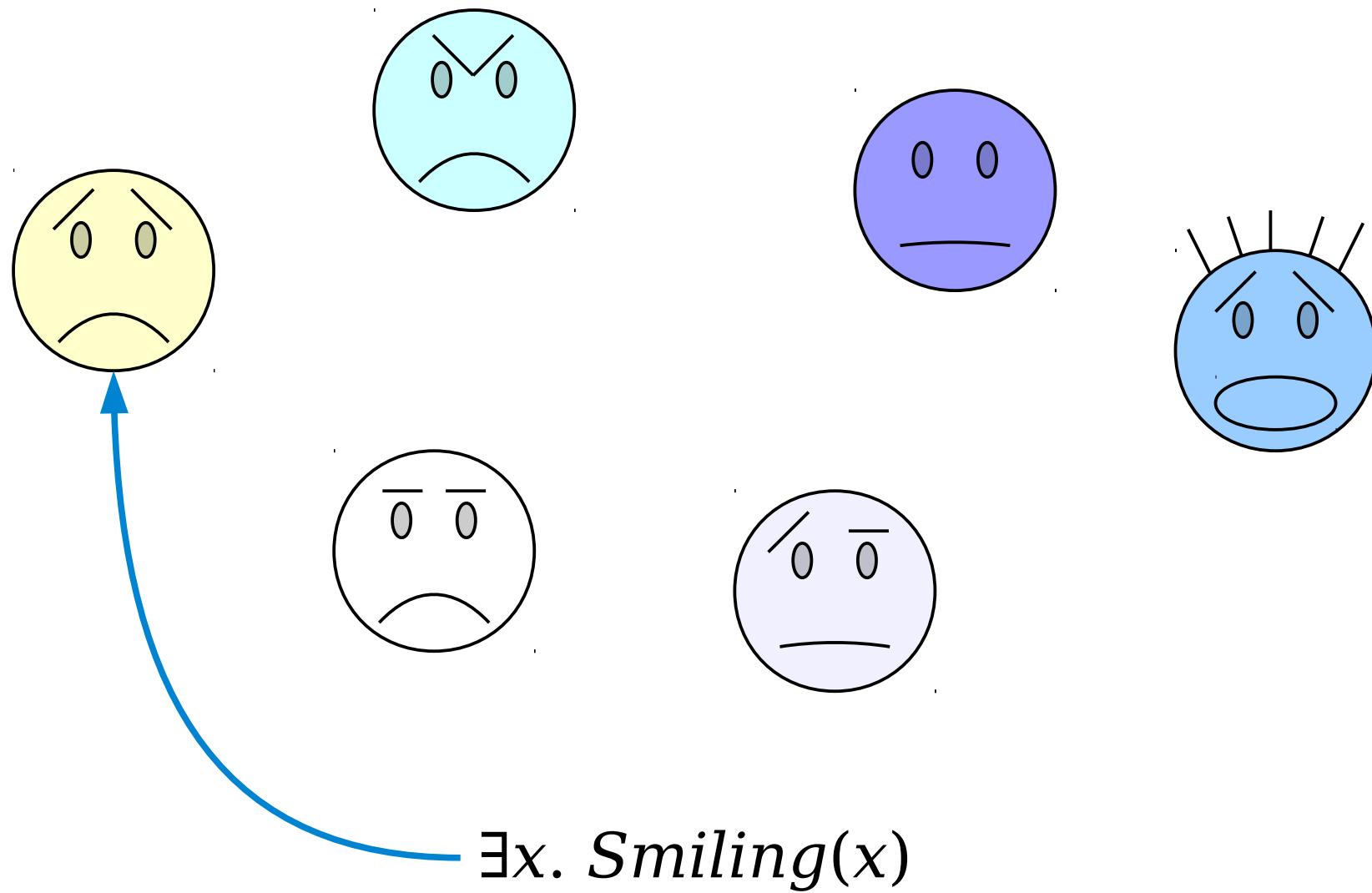


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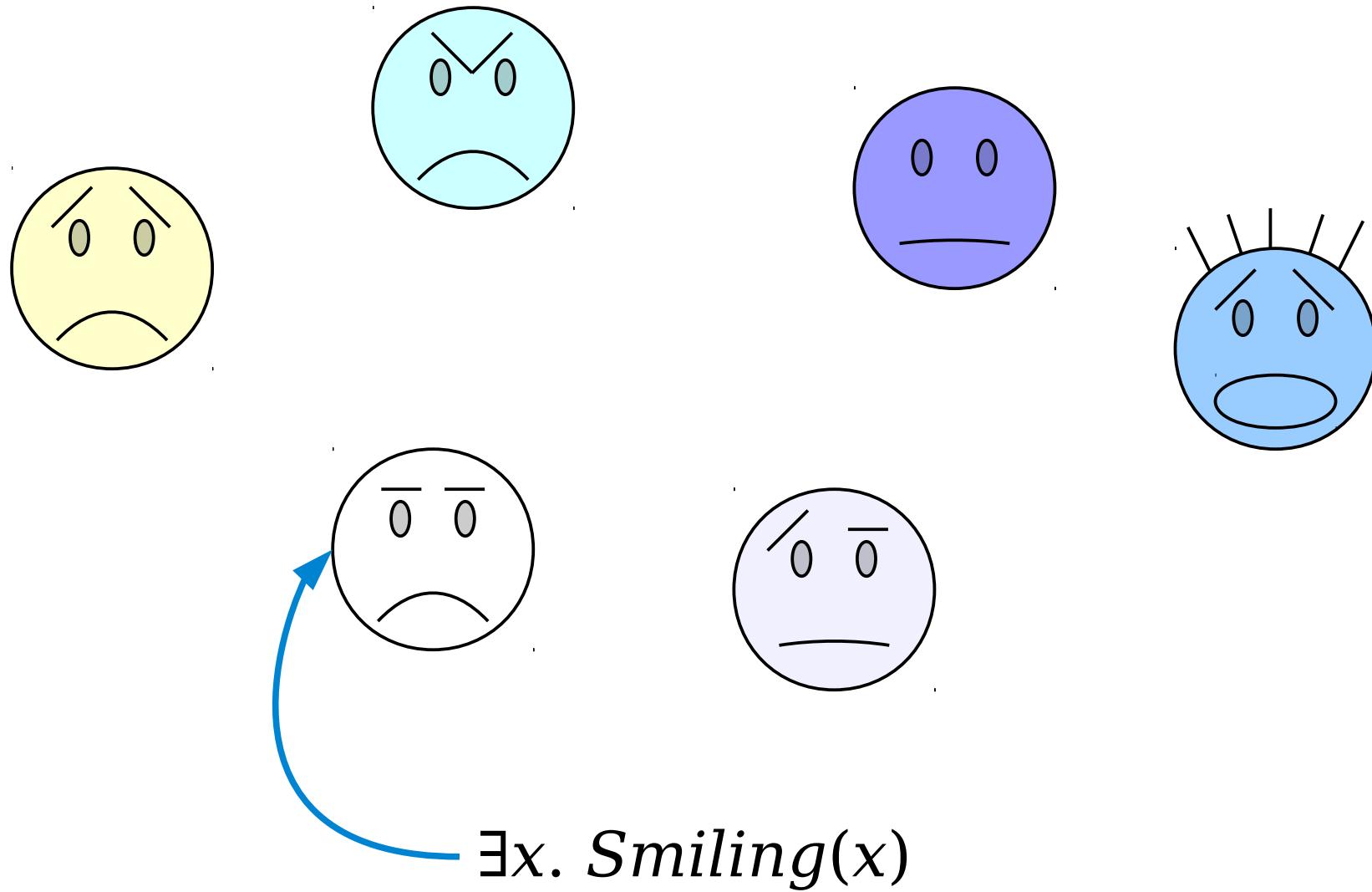


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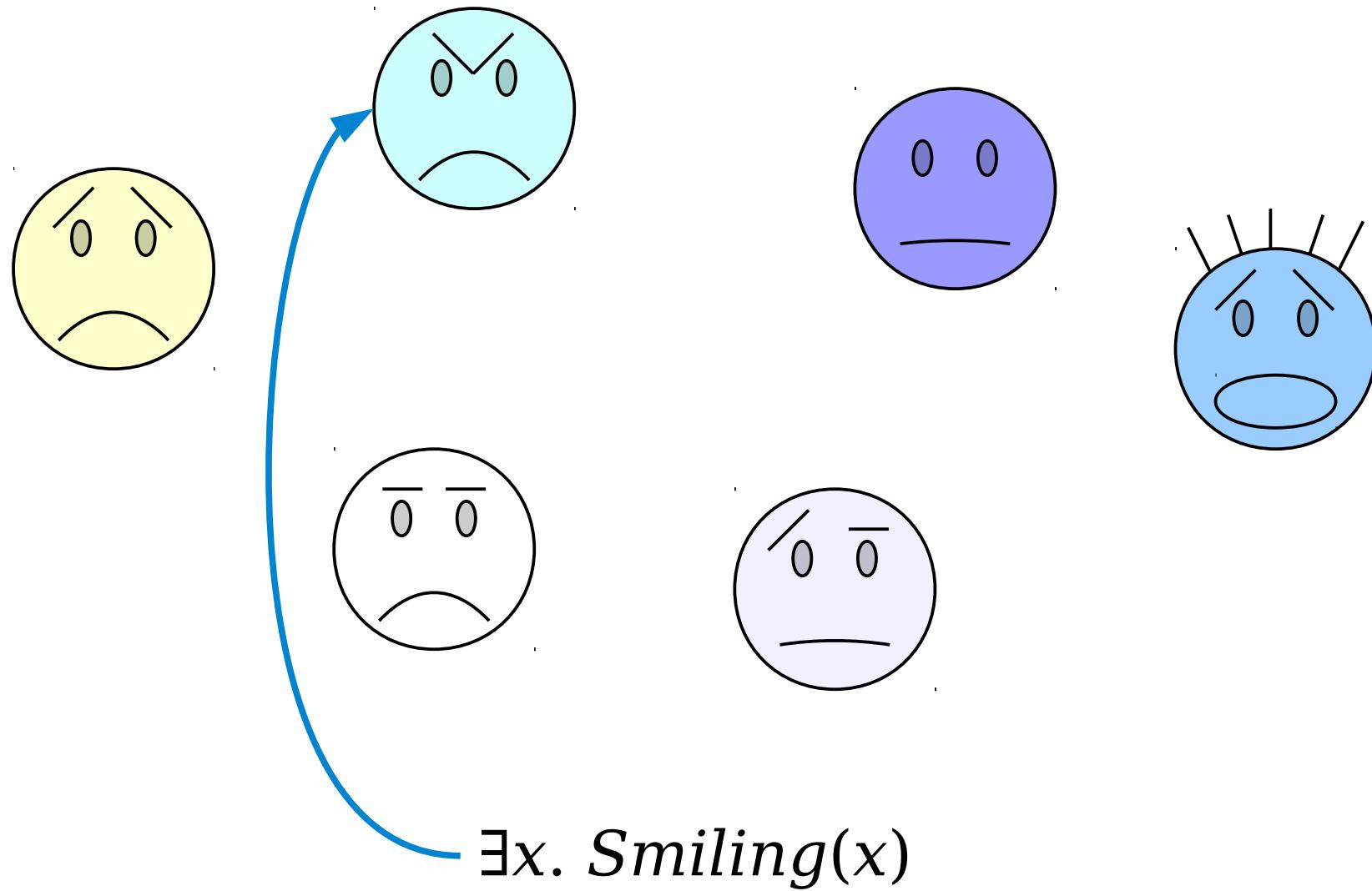
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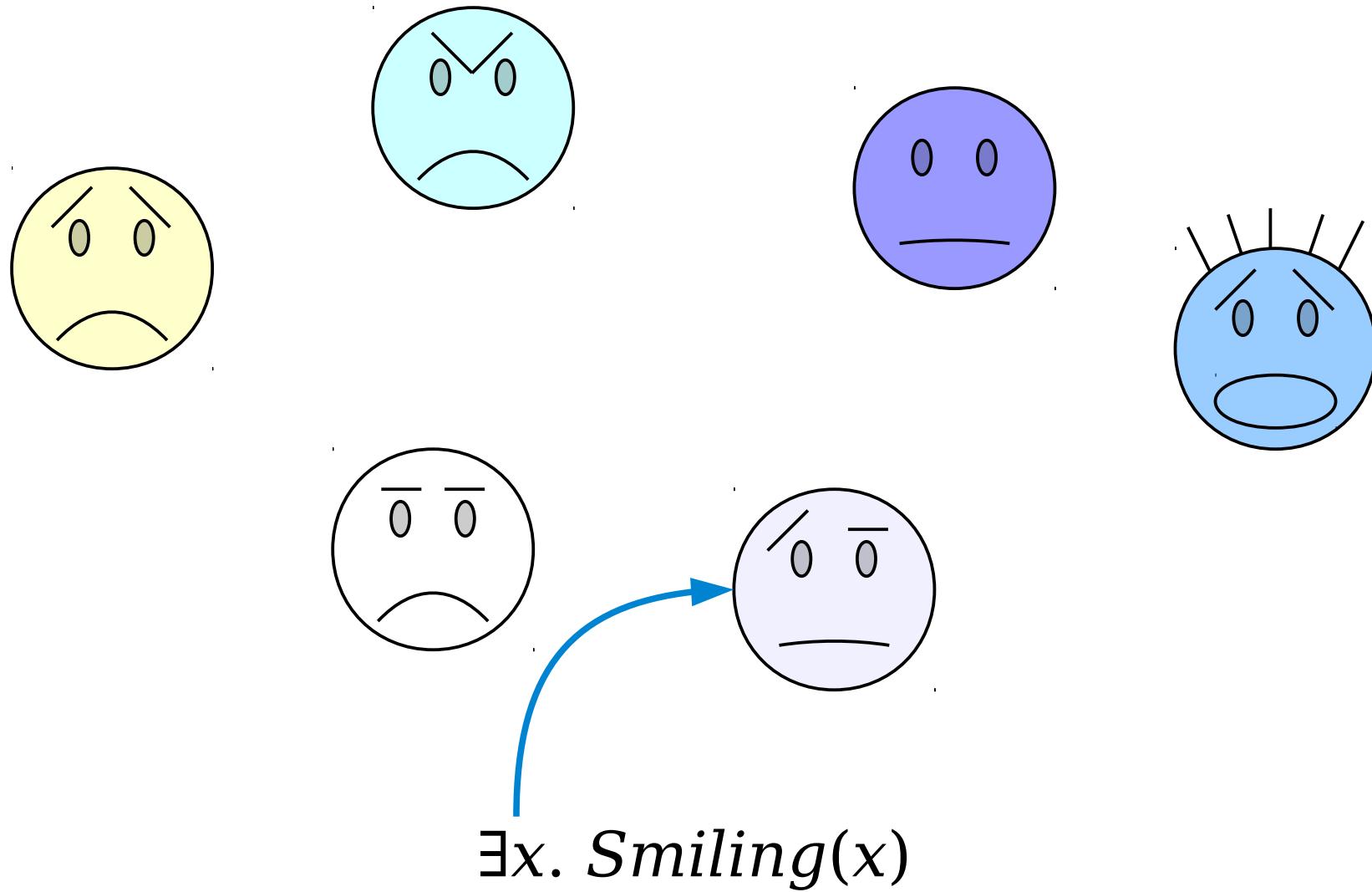
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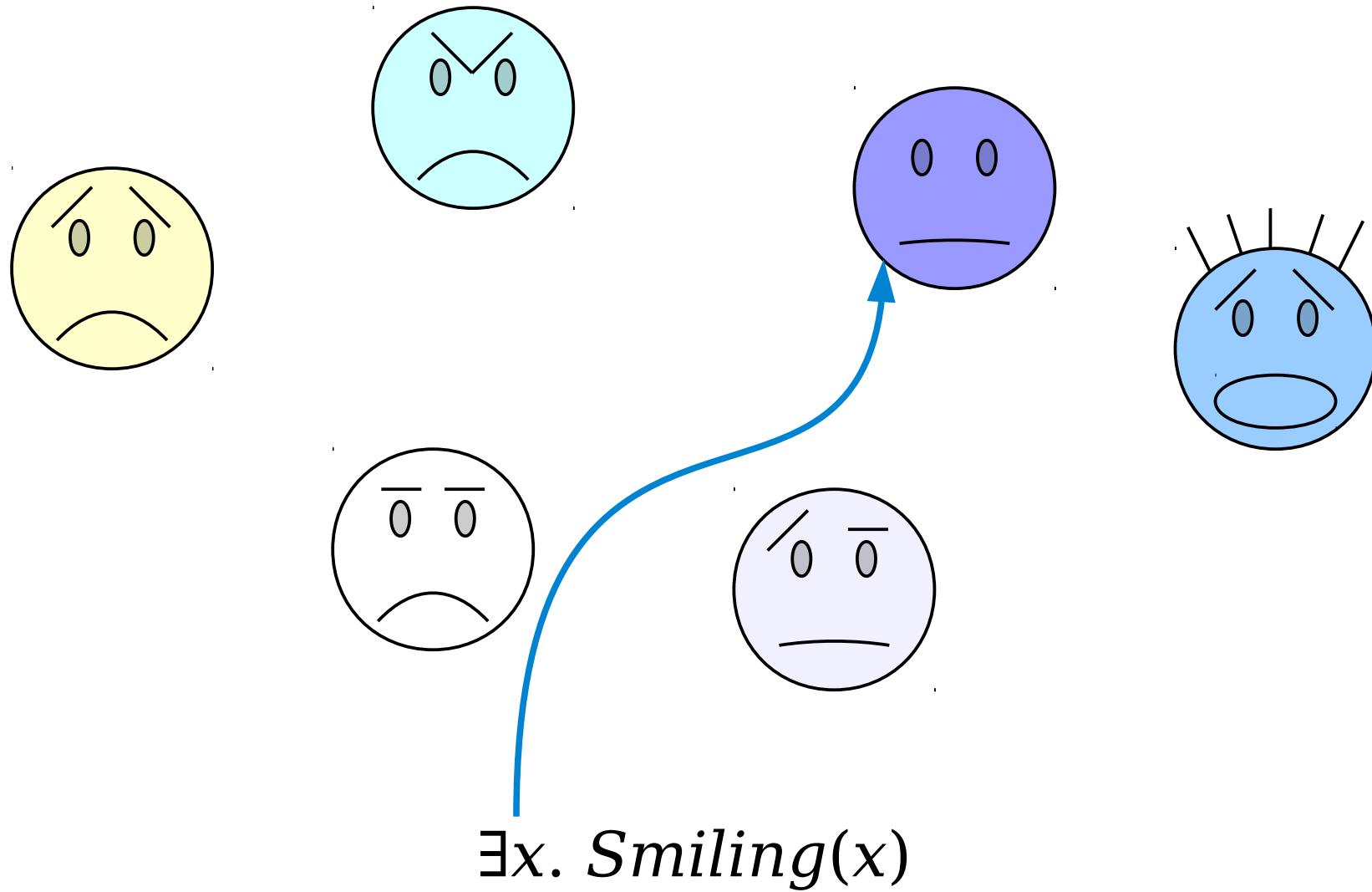
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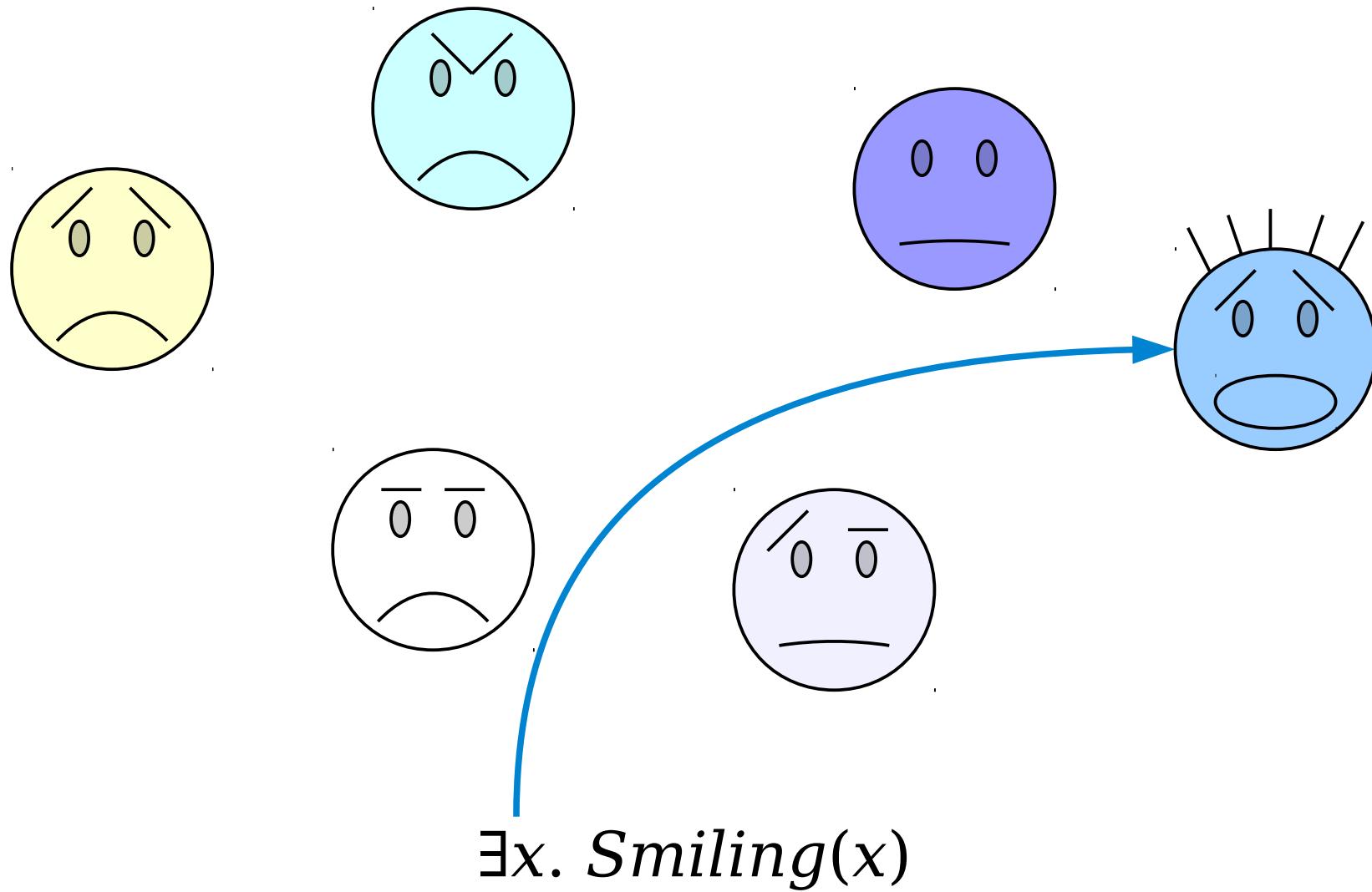
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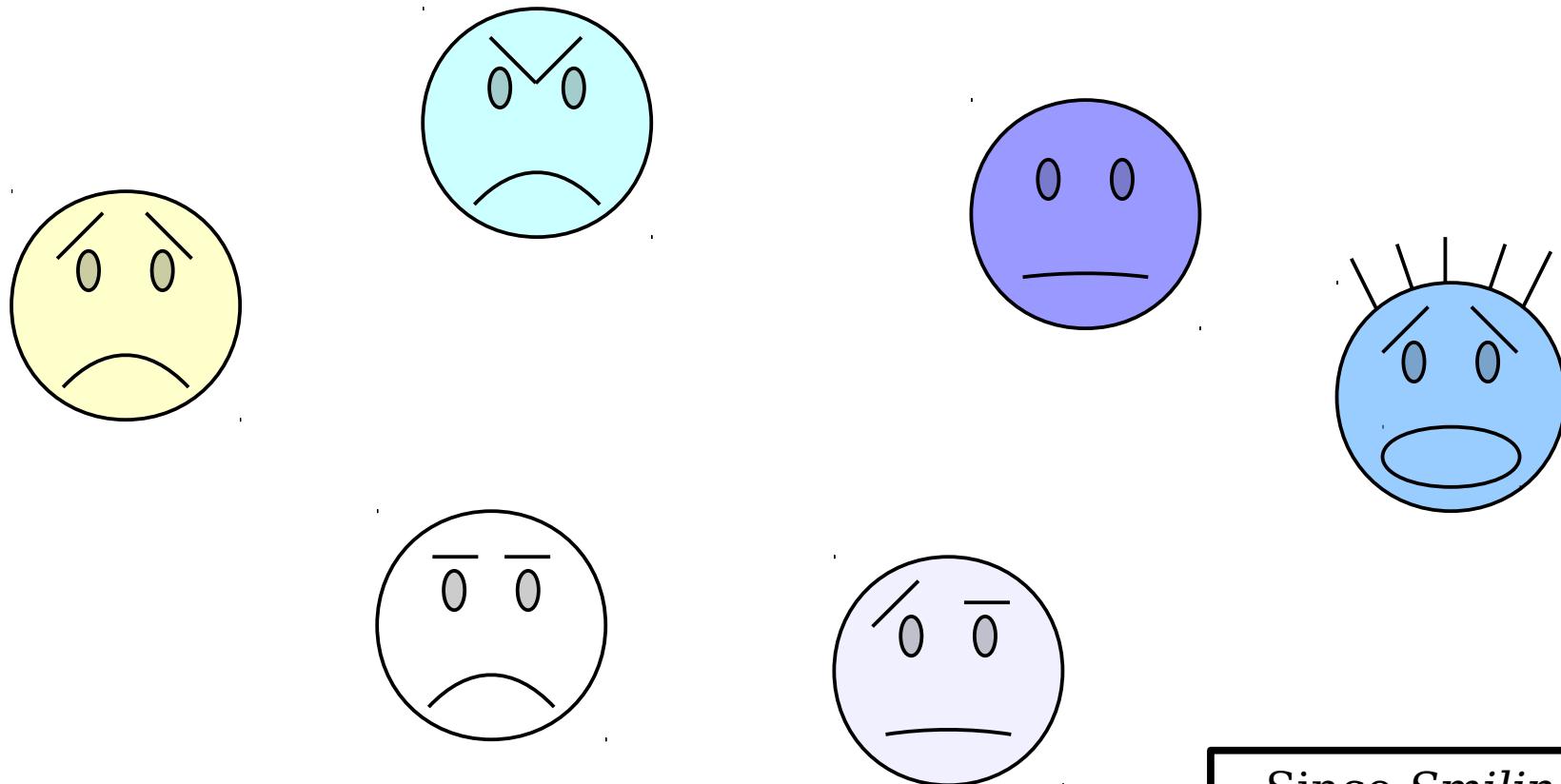
The Existential Quantifier



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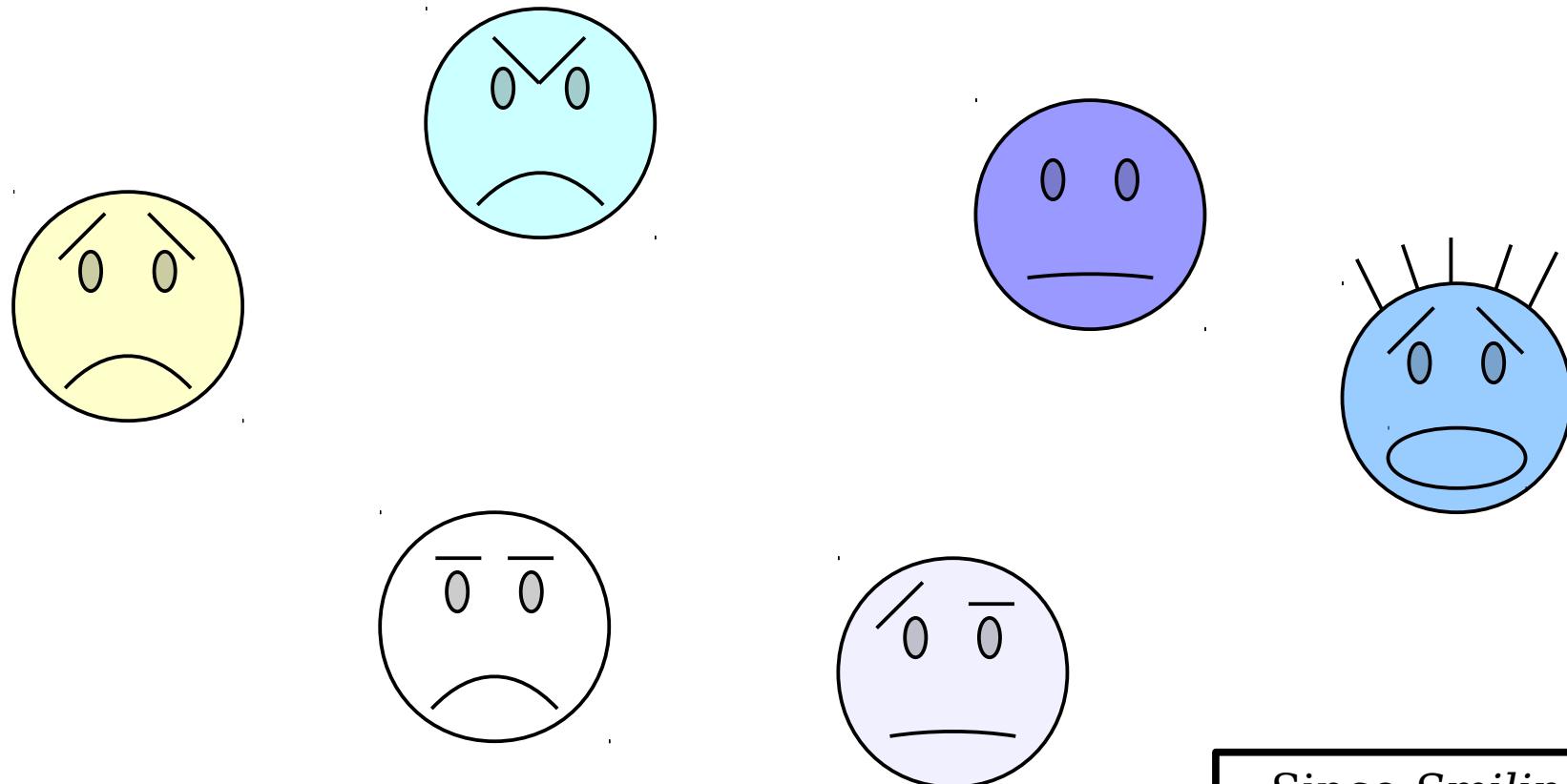
The Existential Quantifier



$\exists x. Smiling(x)$

Since $Smiling(x)$ is not true for any choice of x , this statement evaluates to false.

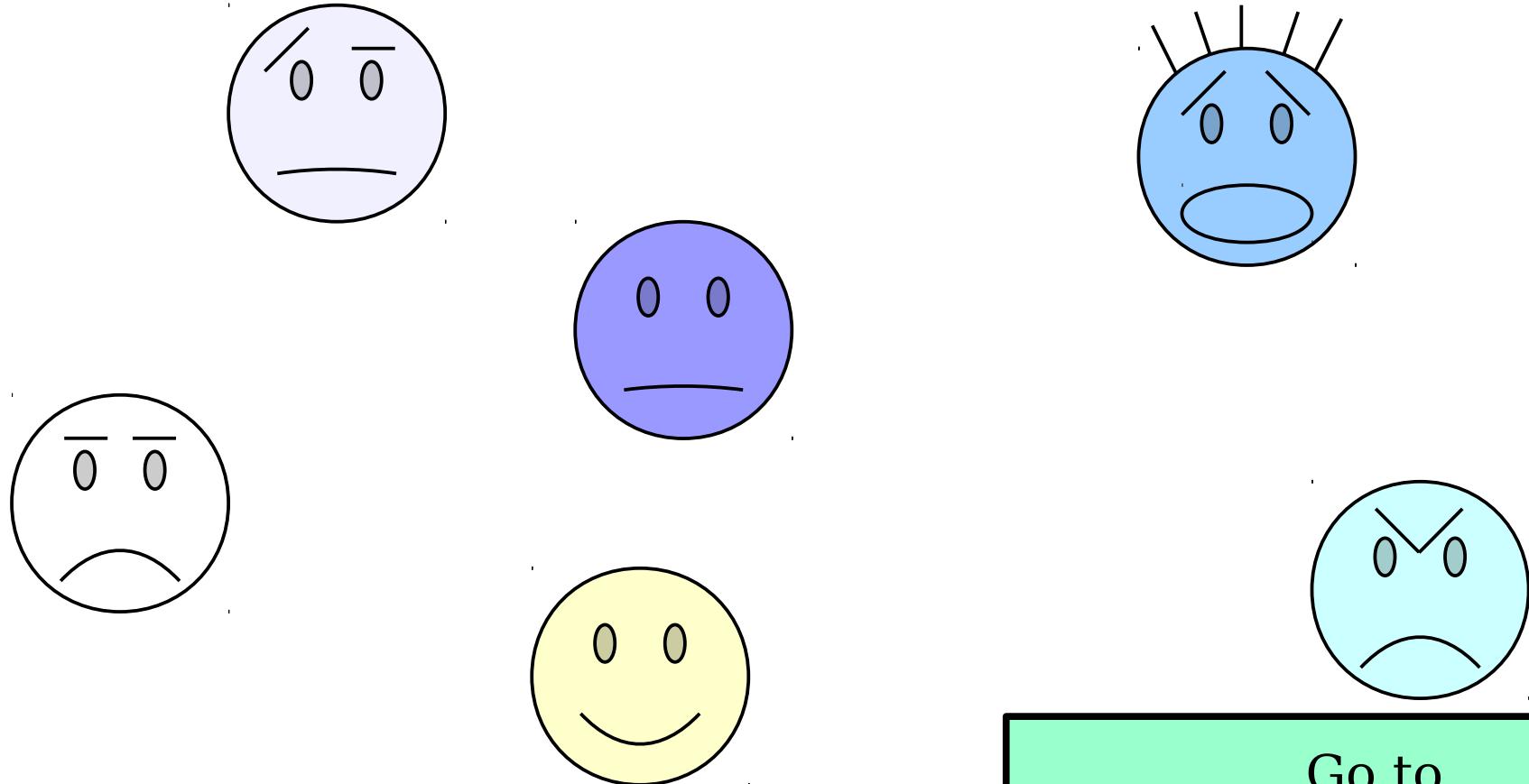
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$\exists x. \text{Smiling}(x)$

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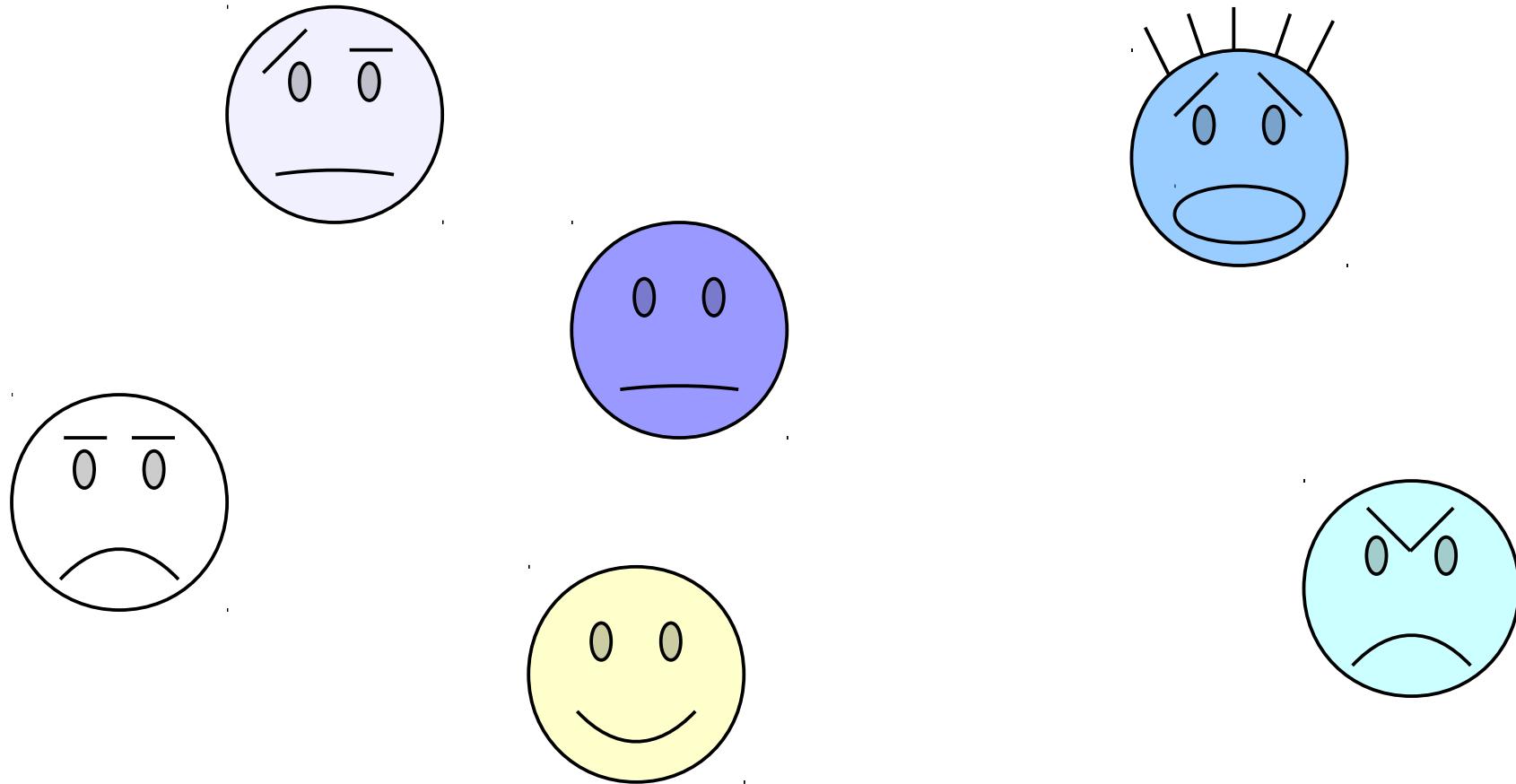
The Existential Quantifier



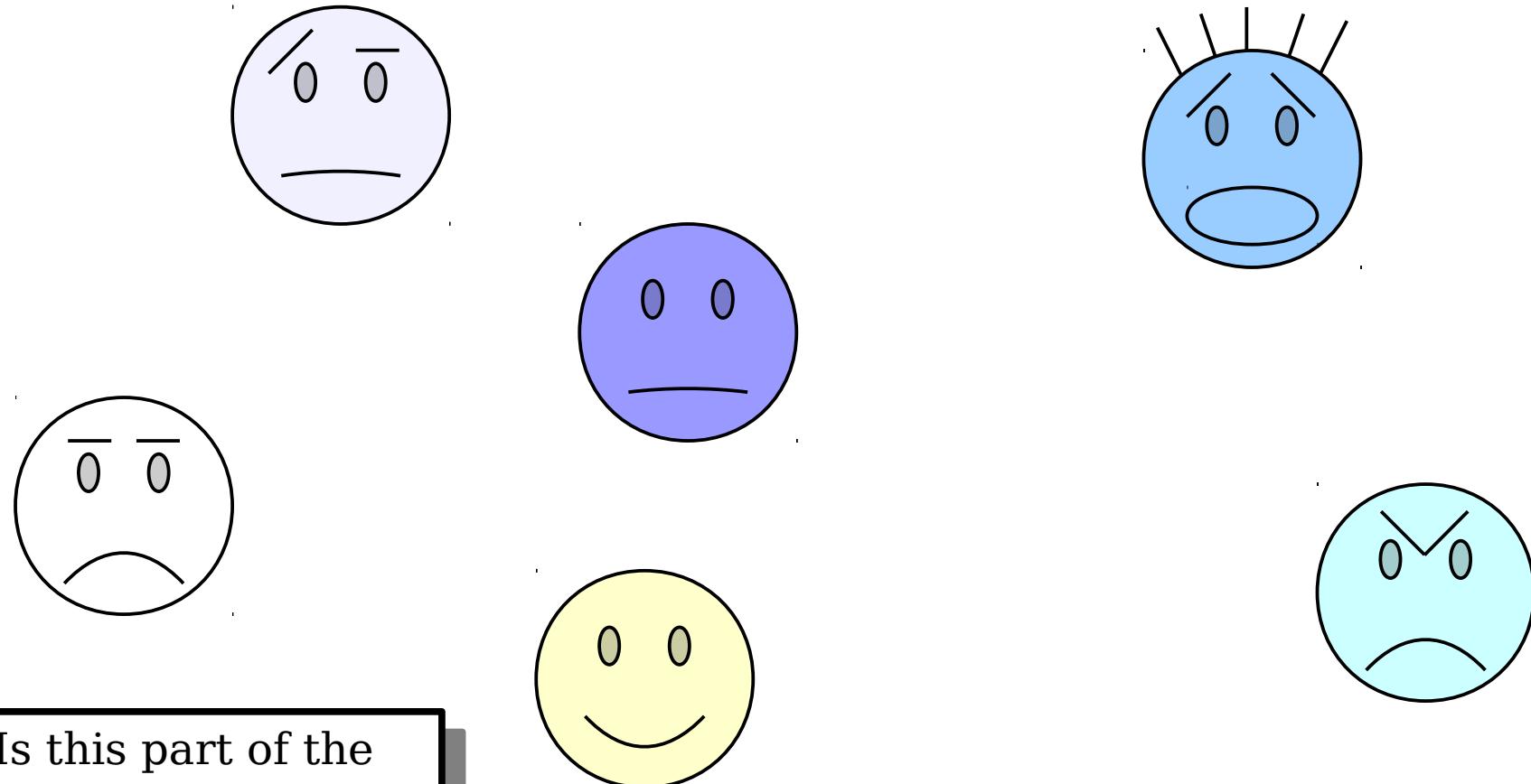
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$(\exists x. Smiling(x)) \rightarrow (\exists y. WearingHat(y))$

The Existential Quantifier


$$(\exists x. Smiling(x)) \rightarrow (\exists y. WearingHat(y))$$

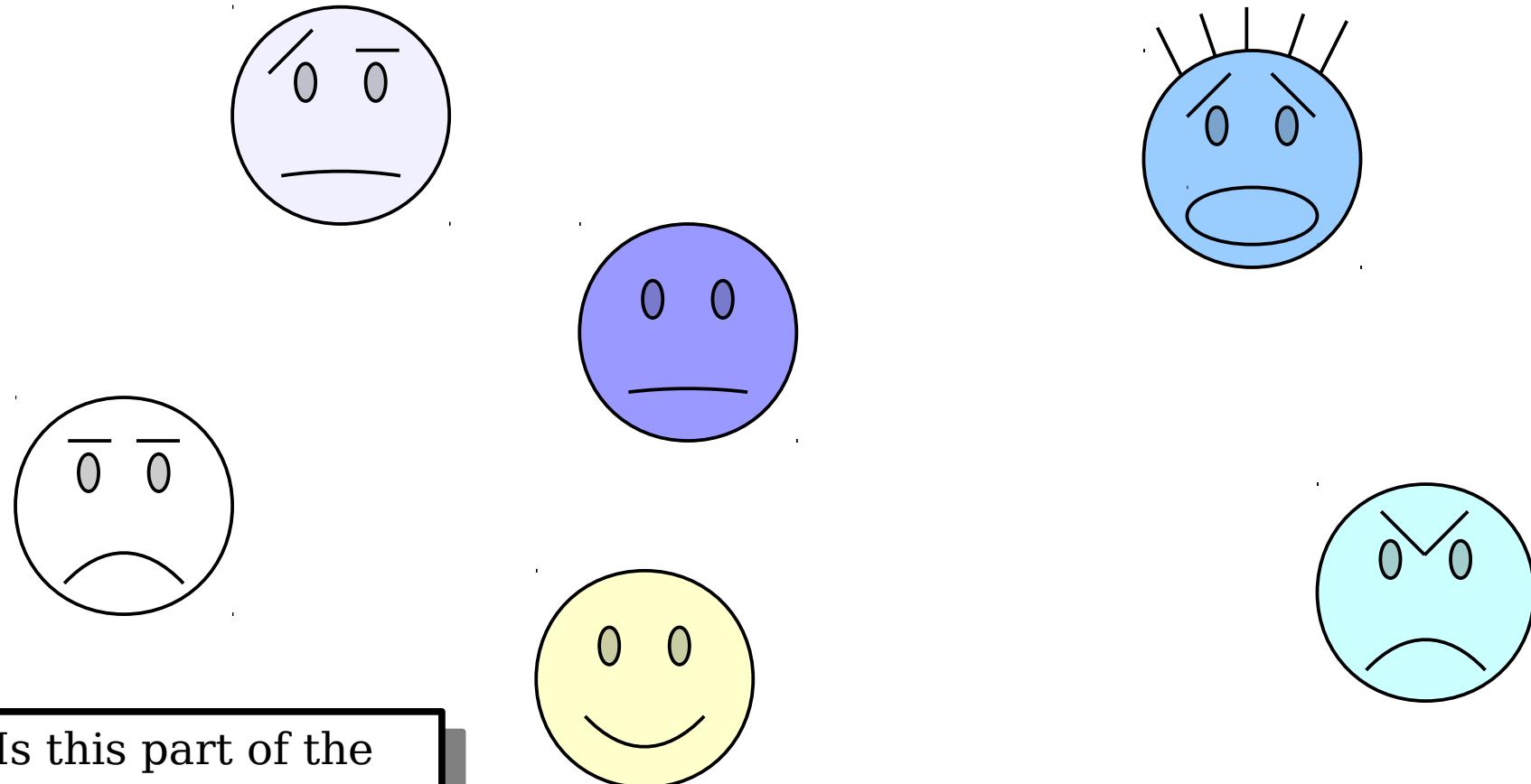
The Existential Quantifier



Is this part of the statement true or false?

$(\exists x. Smiling(x)) \rightarrow (\exists y. WearingHat(y))$

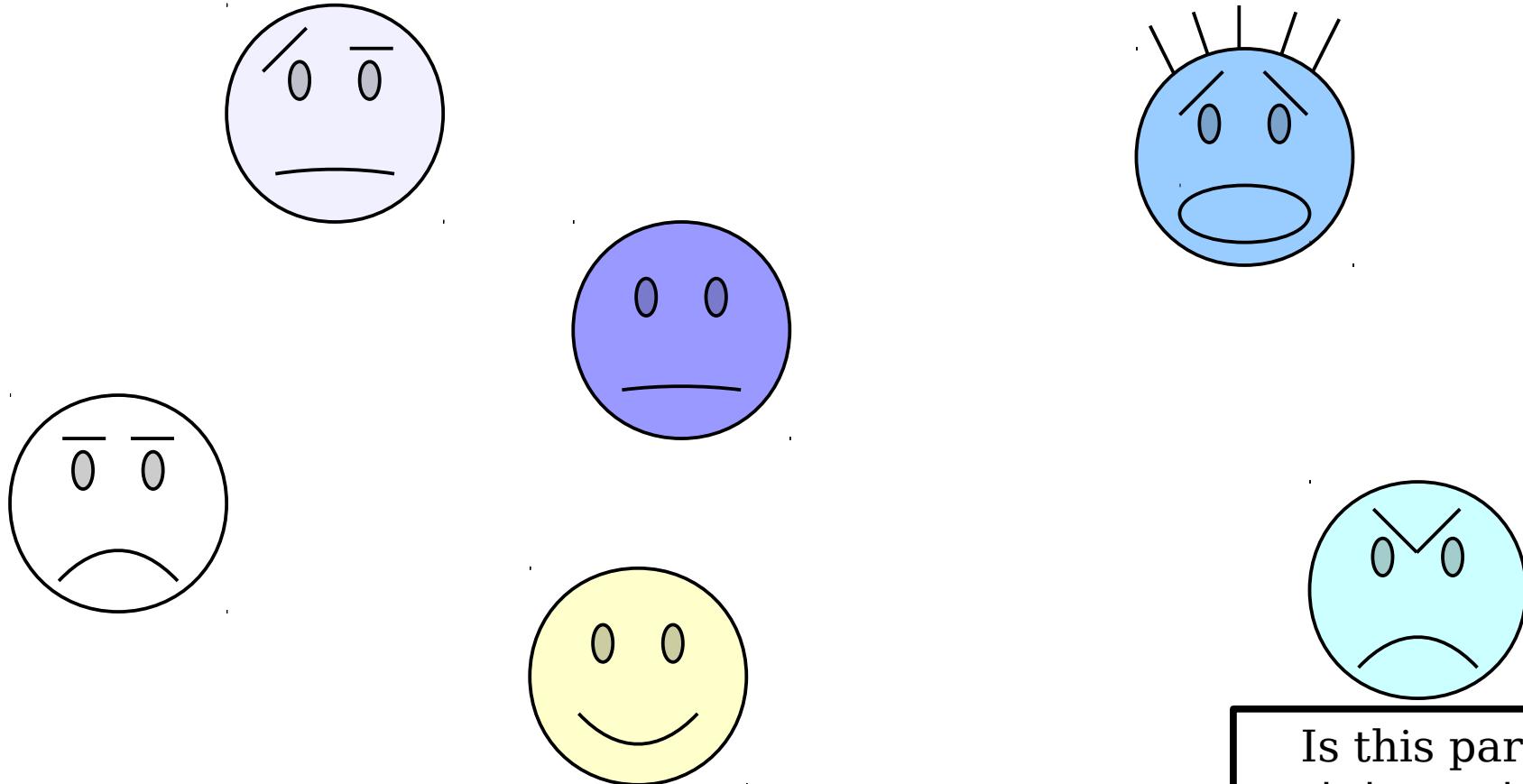
The Existential Quantifier



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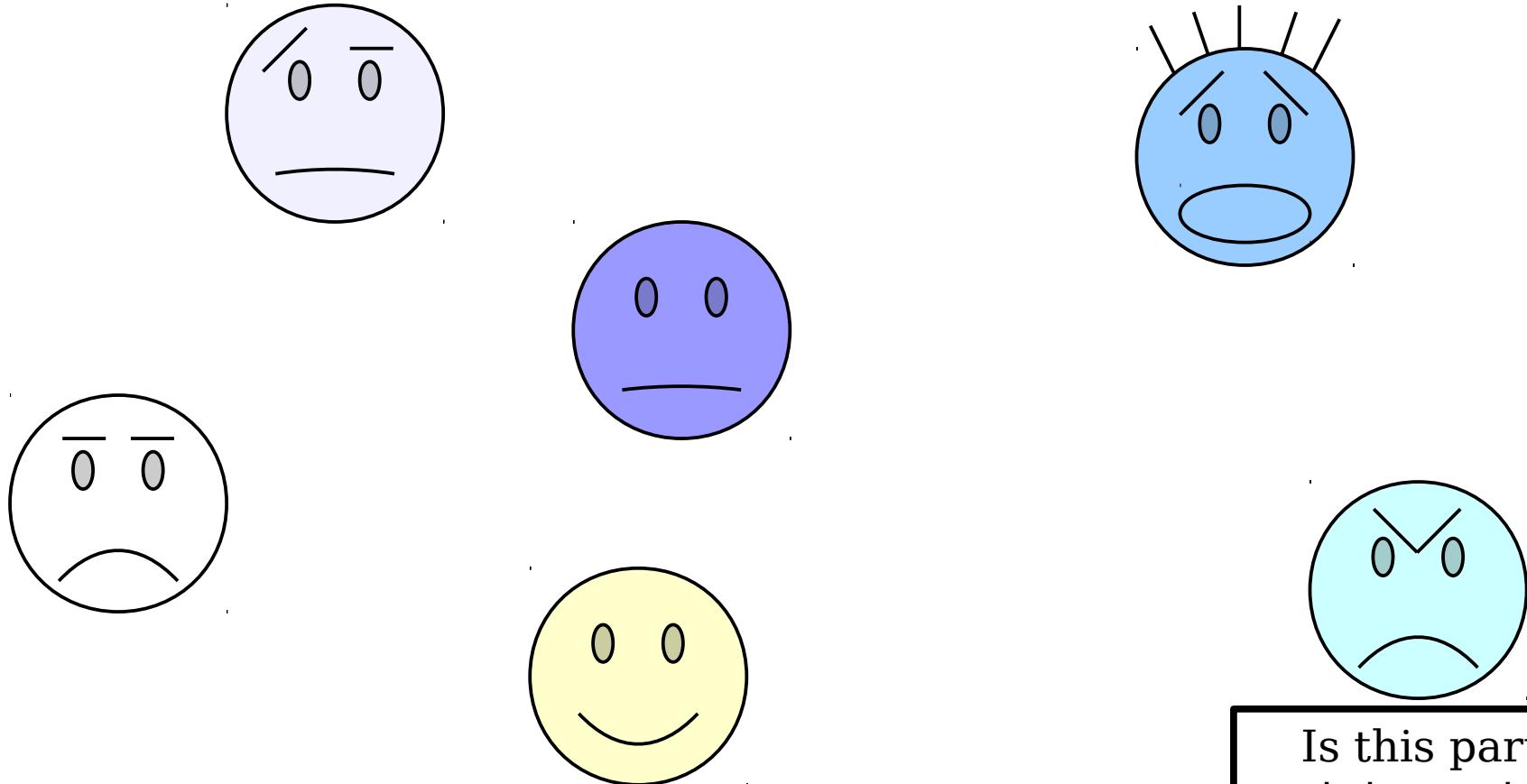
The Existential Quantifier



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$(\exists x. Smiling(x)) \rightarrow (\exists y. WearingHat(y))$

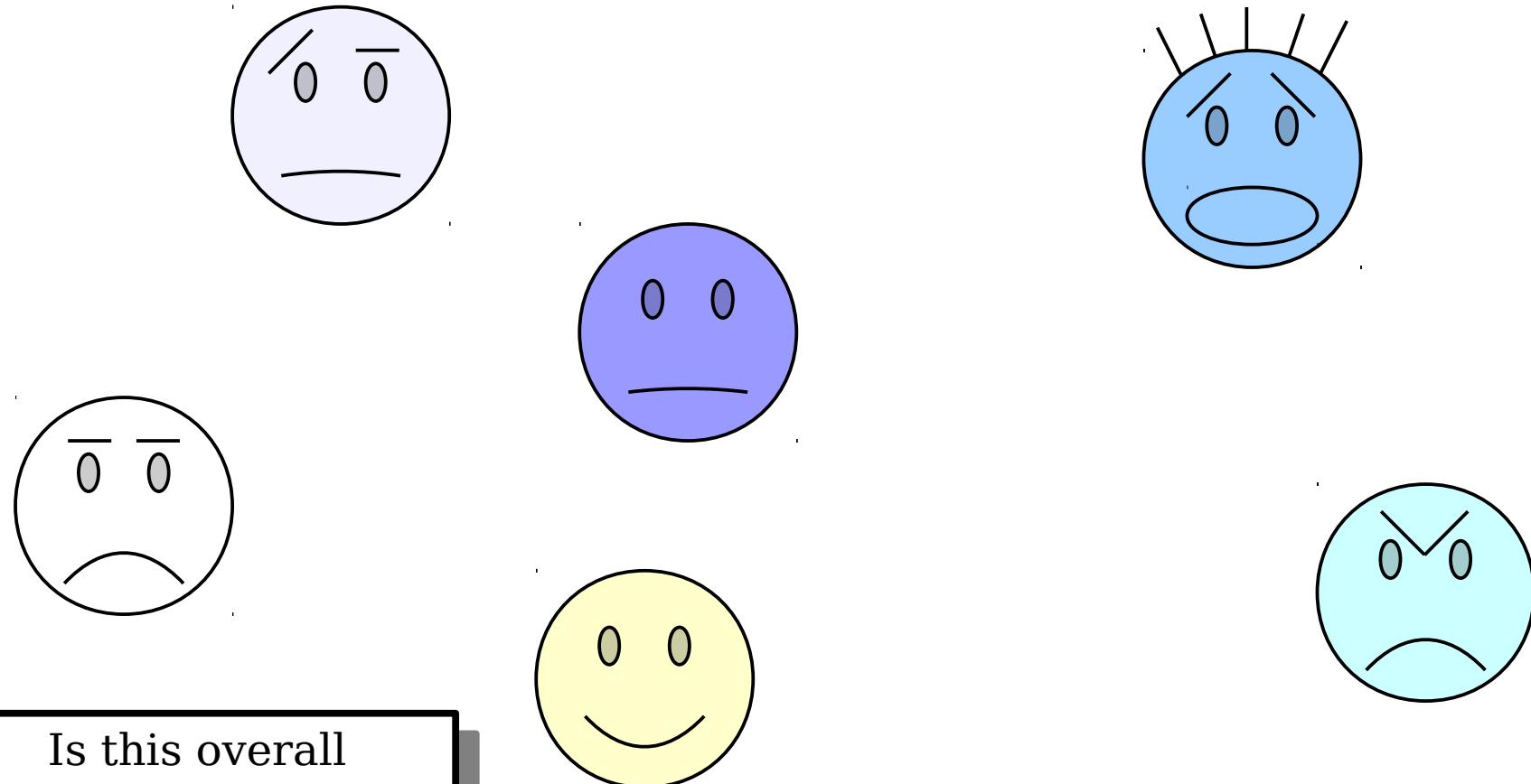
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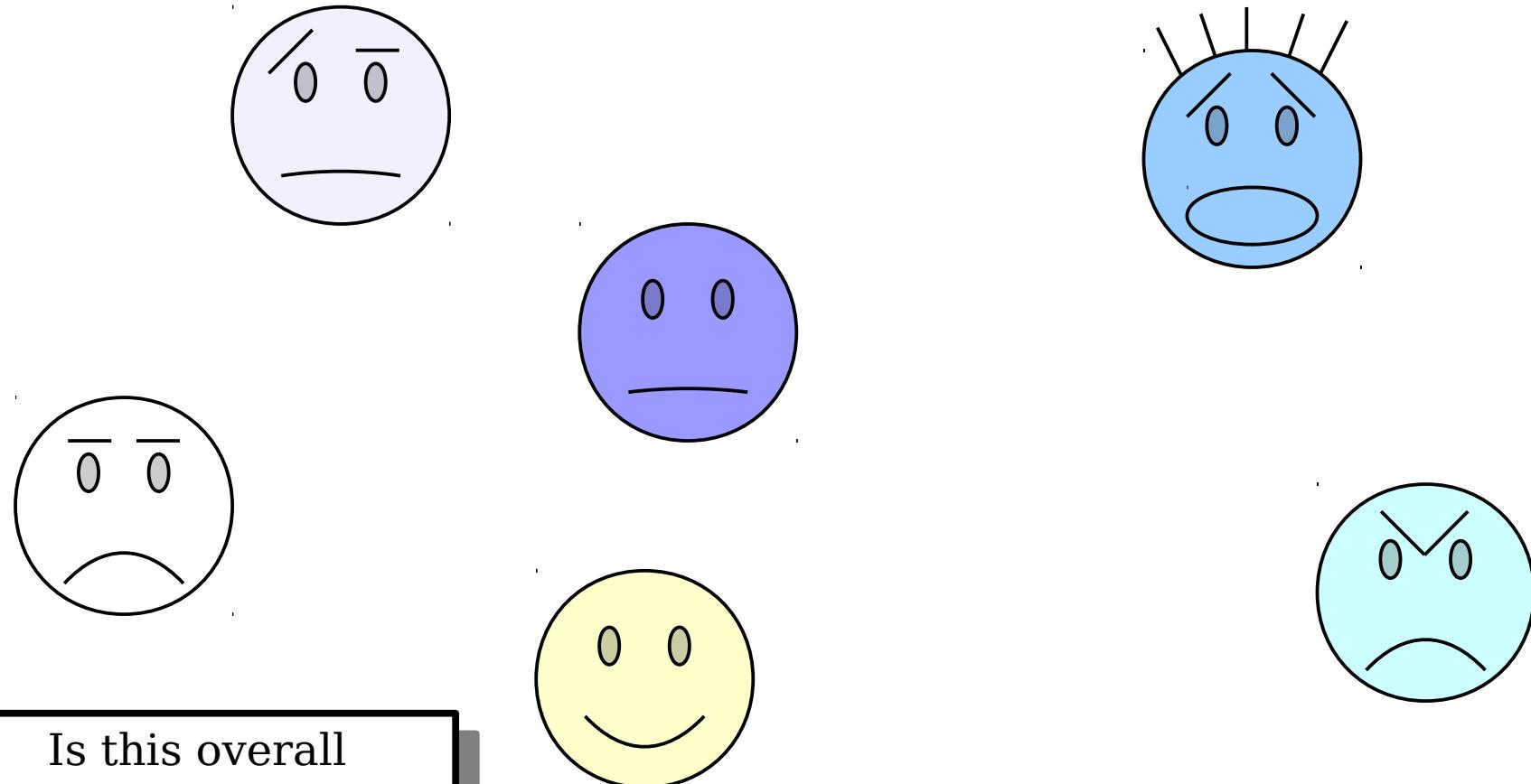
The Existential Quantifier



Is this overall statement true or false?

$$(\exists x. Smiling(x)) \rightarrow (\exists y. WearingHat(y))$$

The Existential Quantifier



Is this overall statement true or false?

$(\exists x. Smiling(x)) \rightarrow (\exists y. WearingHat(y))$

Fun with Edge Cases

$\exists x. Smiling(x)$

Fun with Edge Cases

Existentially-quantified statements are false in an empty world, since nothing exists, period!

$\exists x. Smiling(x)$

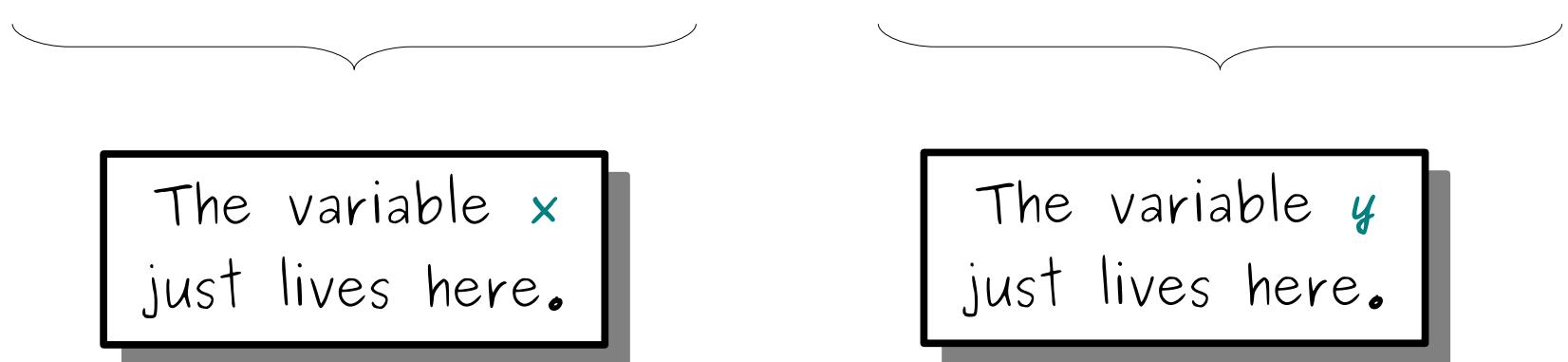
Some Technical Details

Variables and Quantifiers

- Each quantifier has two parts:
 - the variable that is introduced, and
 - the statement that's being quantified.
- The variable introduced is scoped just to the statement being quantified.
$$(\exists x. Loves(You, x)) \wedge (\exists y. Loves(y, You))$$

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The variable x
just lives here.

The variable y
just lives here.

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$$(\exists x. Loves(You, x)) \wedge (\exists x. Loves(x, You))$$

The variable x
just lives here.

A different variable,
also named x , just
lives here.

Operator Precedence (Again)

- When writing out a formula in first-order logic, quantifiers have precedence just below \neg .
- The statement

$$\exists x. P(x) \wedge R(x) \wedge Q(x)$$

is parsed like this:

$$\square (\exists \textcolor{blue}{x}. P(\textcolor{blue}{x})) \wedge (R(\textcolor{red}{x}) \wedge Q(\textcolor{red}{x})) \square$$

- This is syntactically invalid because the variable x is out of scope in the back half of the formula.
- To ensure that x is properly quantified, explicitly put parentheses around the region you want to quantify:

$$\exists x. (P(x) \wedge R(x) \wedge Q(x))$$

“For any natural number n ,
 n is even if and only if n^2 is even”

“For any natural number n ,
 n is even if and only if n^2 is even”

$$\forall n. (n \in \mathbb{N} \rightarrow (Even(n) \leftrightarrow Even(n^2)))$$

“For any natural number n ,
 n is even if and only if n^2 is even”

$\forall n. (n \in \mathbb{N} \rightarrow (Even(n) \leftrightarrow Even(n^2)))$

\forall is the **universal quantifier**
and says “for any choice of n ,
the following is true.”

The Universal Quantifier

- A statement of the form

$\forall x. \text{some-formula}$

is true if, for every choice of x , the statement **some-formula** is true when x is plugged into it.

- Examples:

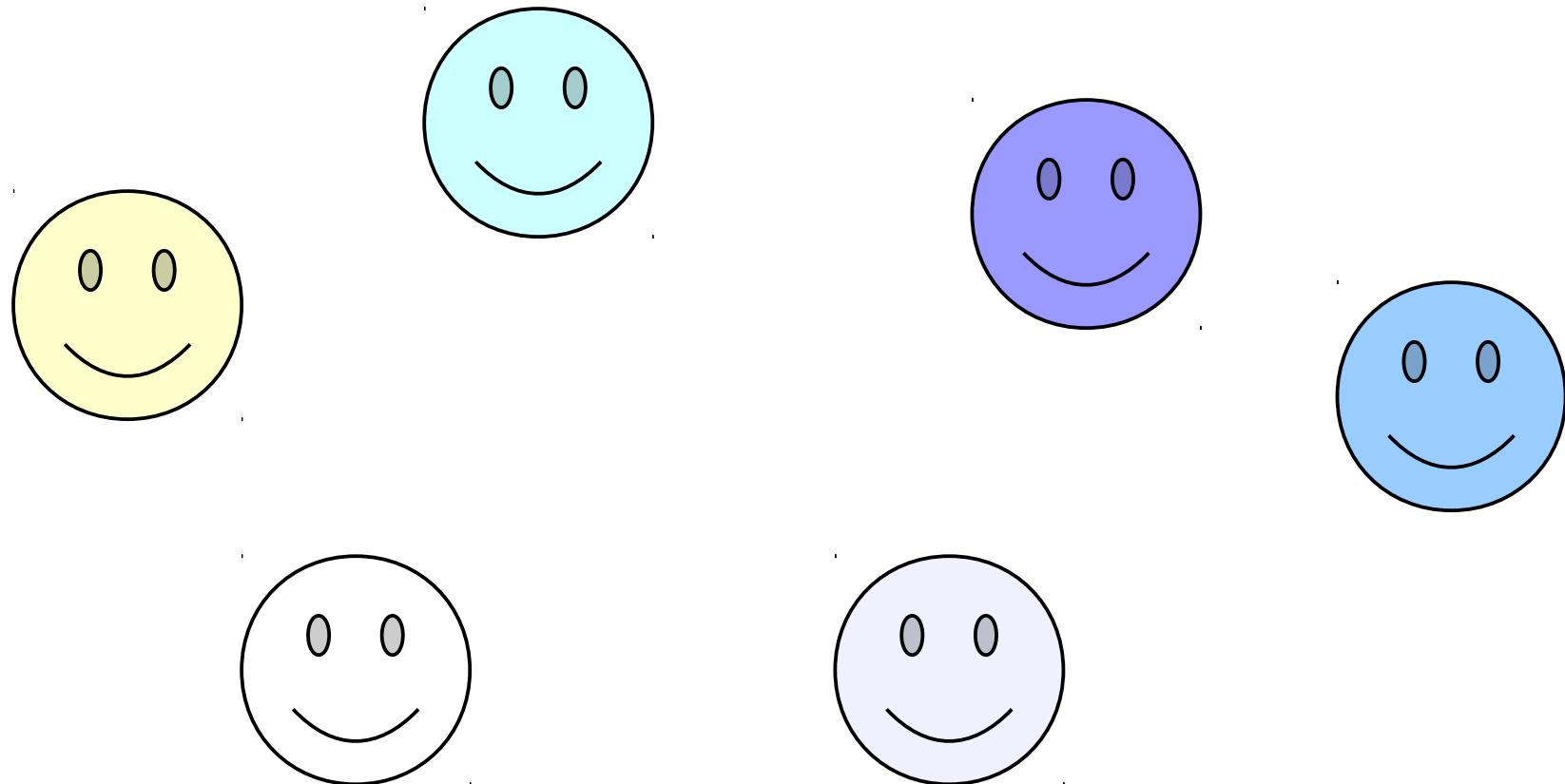
$\forall p. (\text{Puppy}(p) \rightarrow \text{Cute}(p))$

$\forall a. (\text{EatsPlants}(a) \vee \text{EatsAnimals}(a))$

$\text{Tallest}(\text{SultanKösen}) \rightarrow$

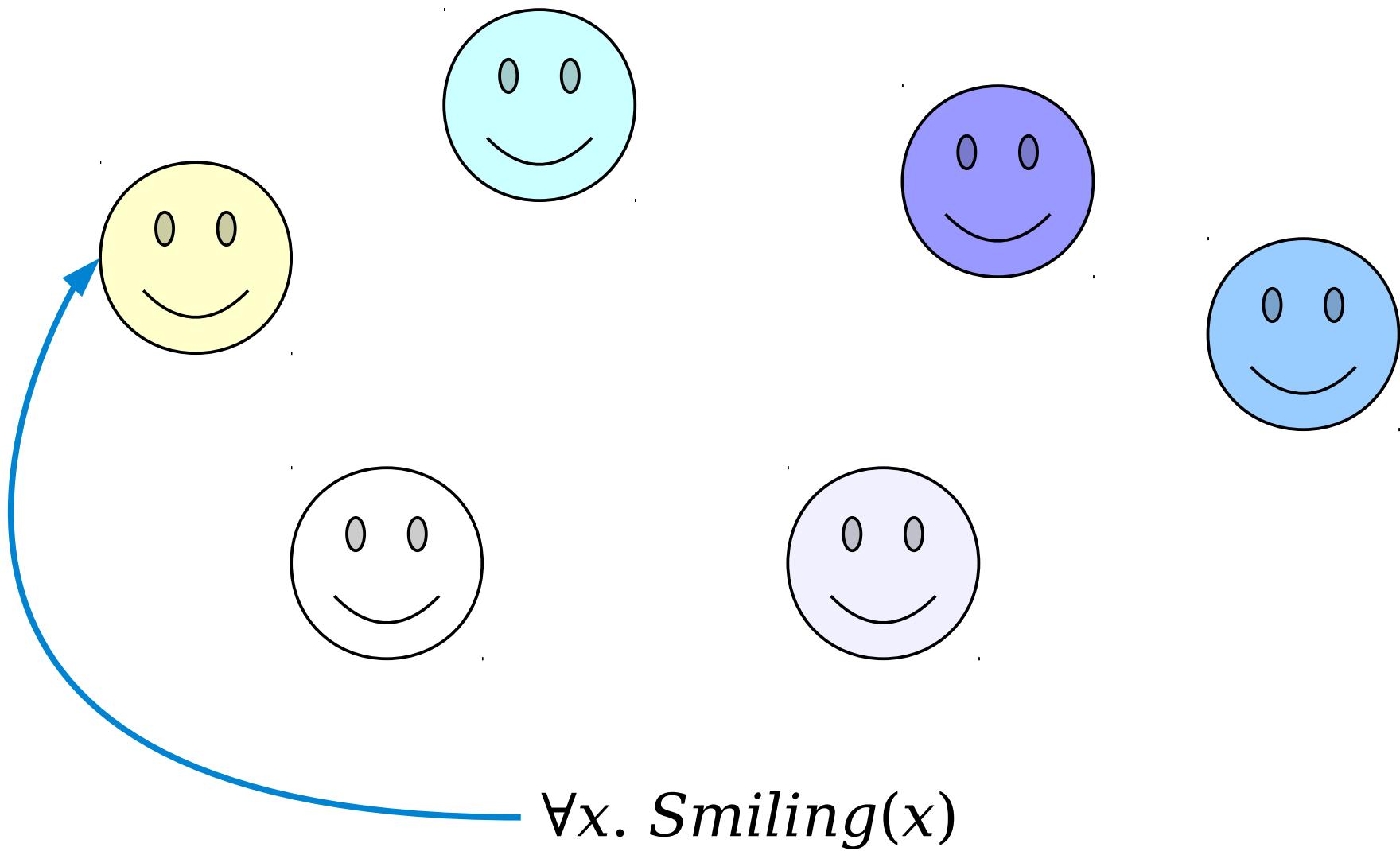
$\forall x. (\text{SultanKösen} \neq x \rightarrow \text{ShorterThan}(x, \text{SultanKösen}))$

The Universal Quantifier

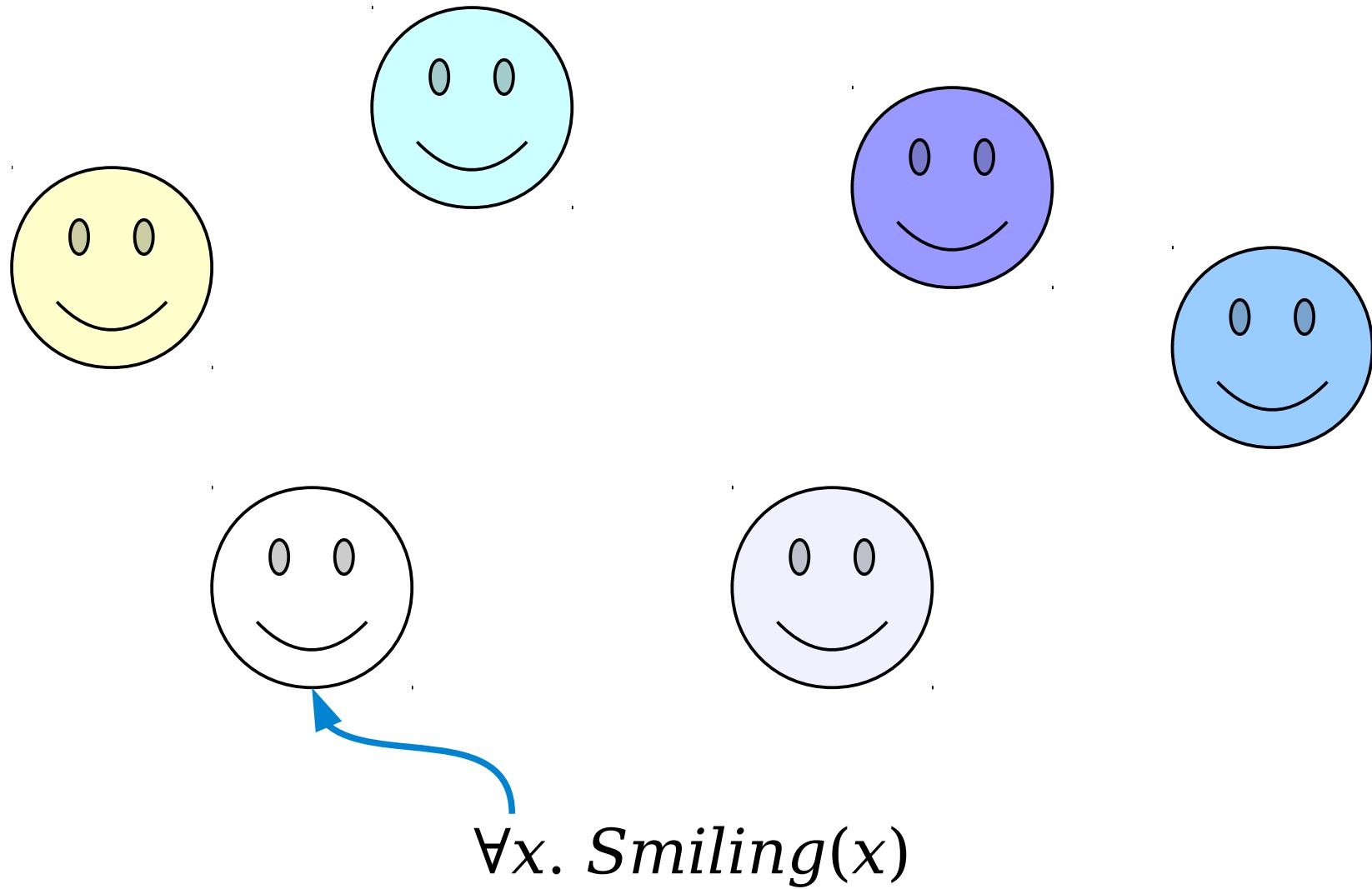


$\forall x. Smiling(x)$

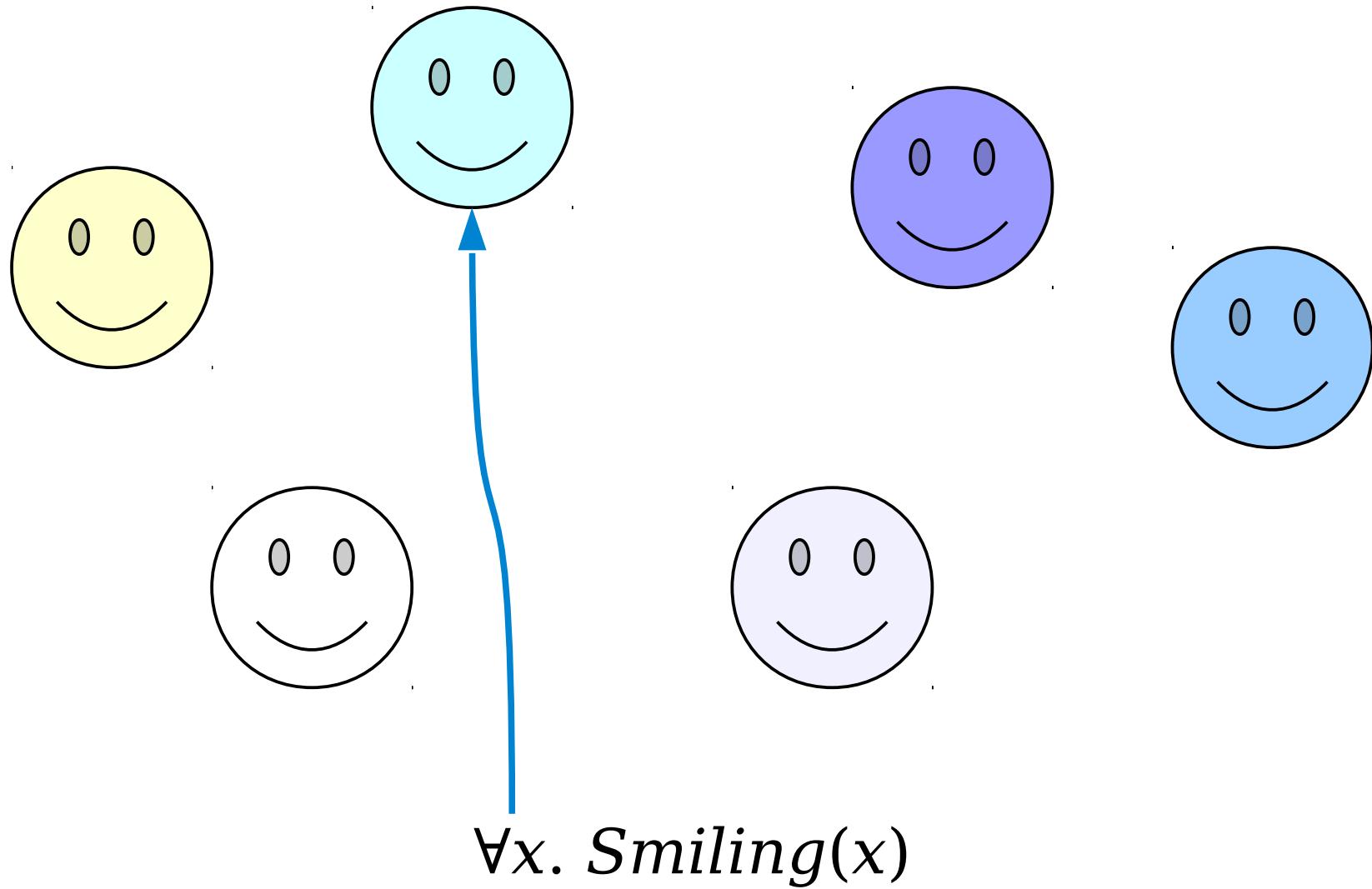
The Universal Quantifier



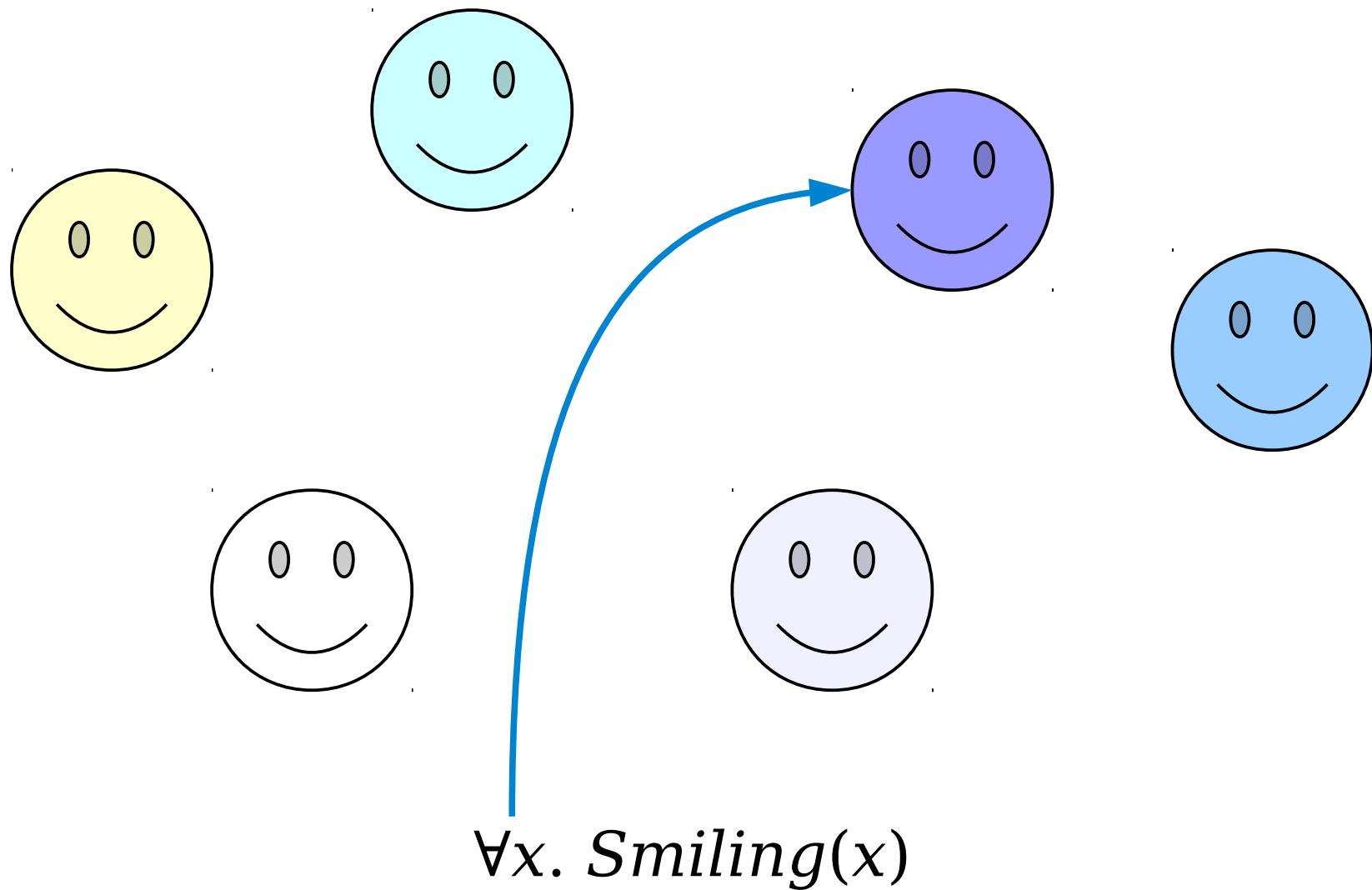
The Universal Quantifier



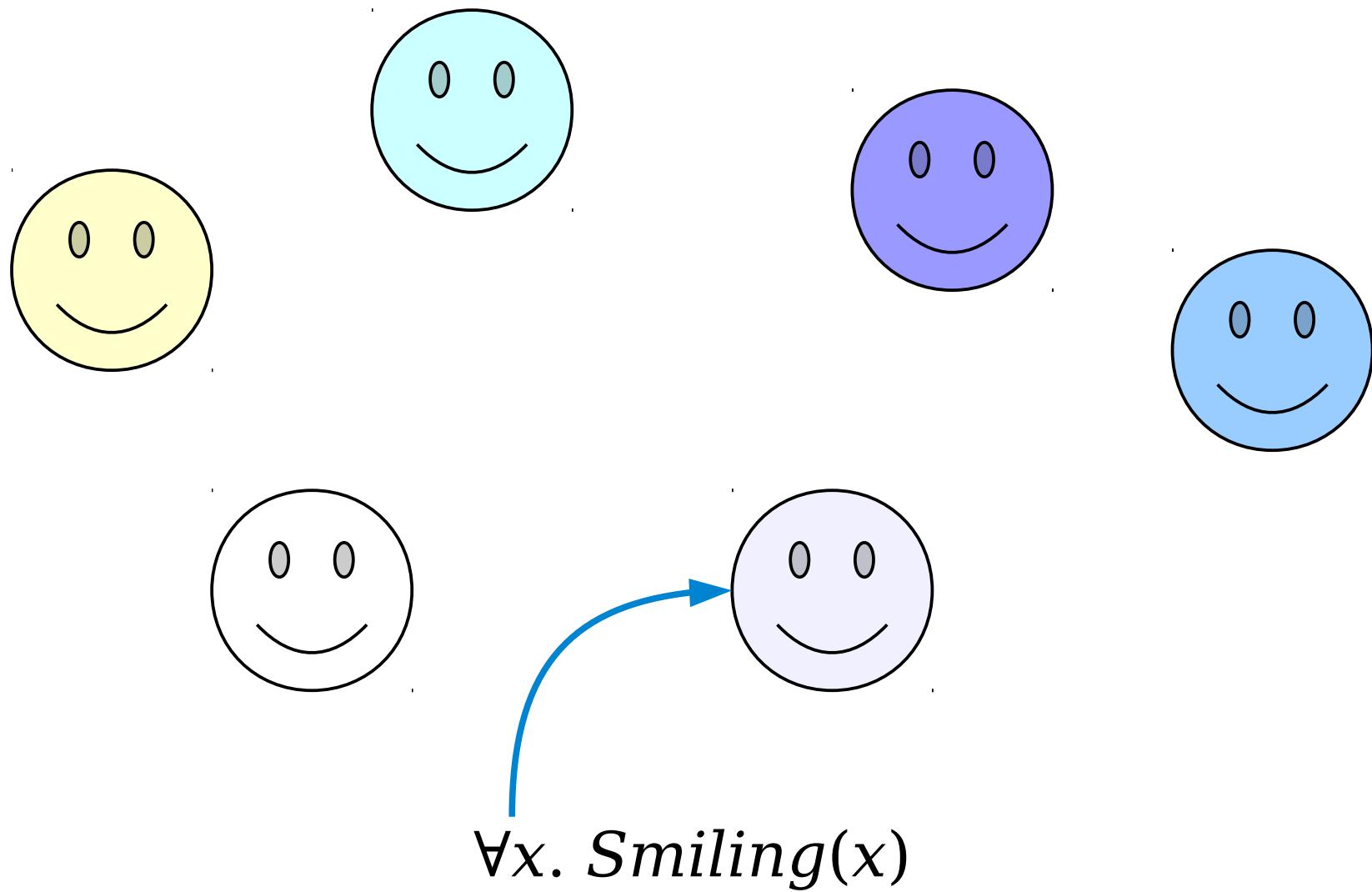
The Universal Quantifier



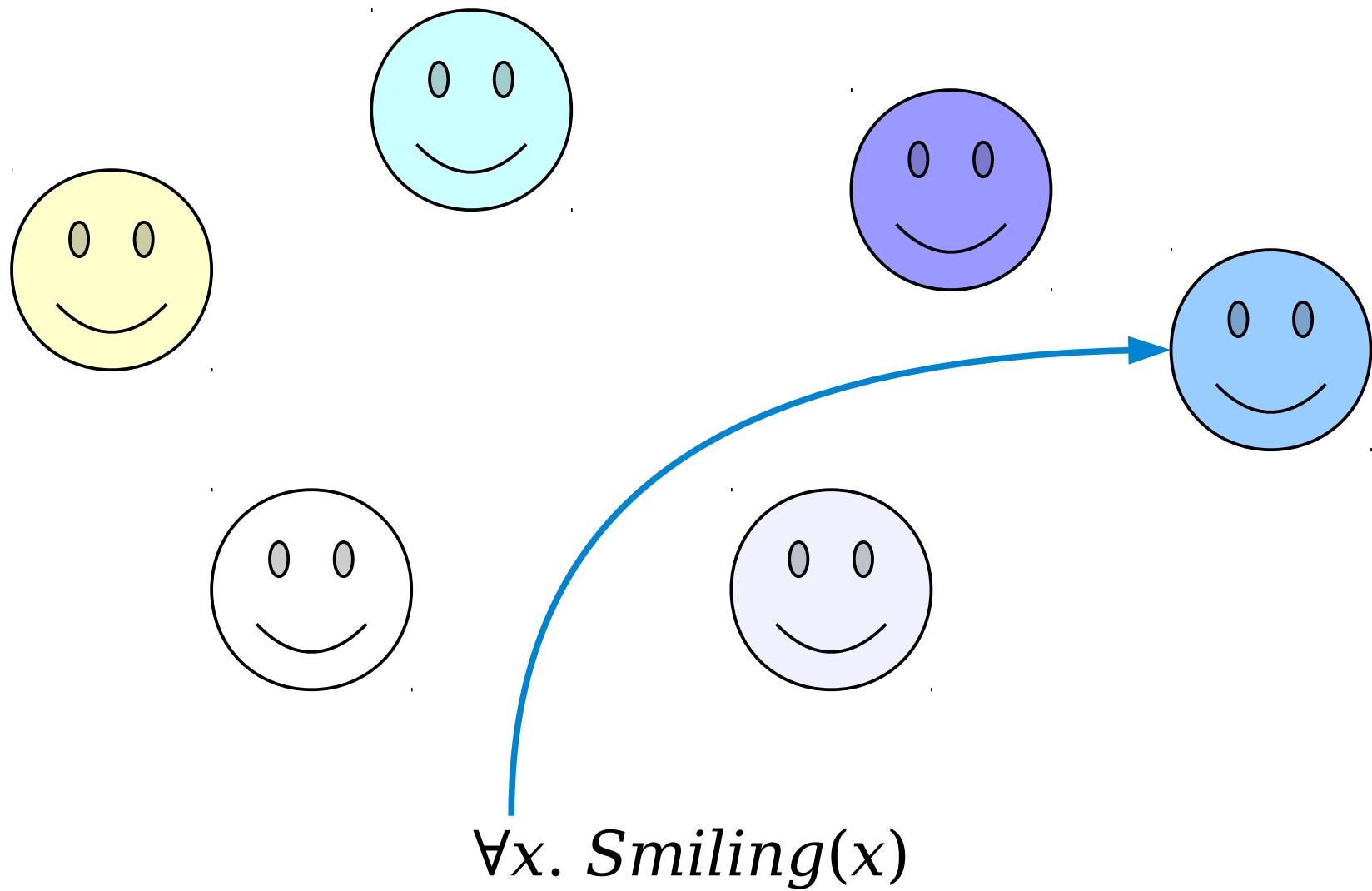
The Universal Quantifier



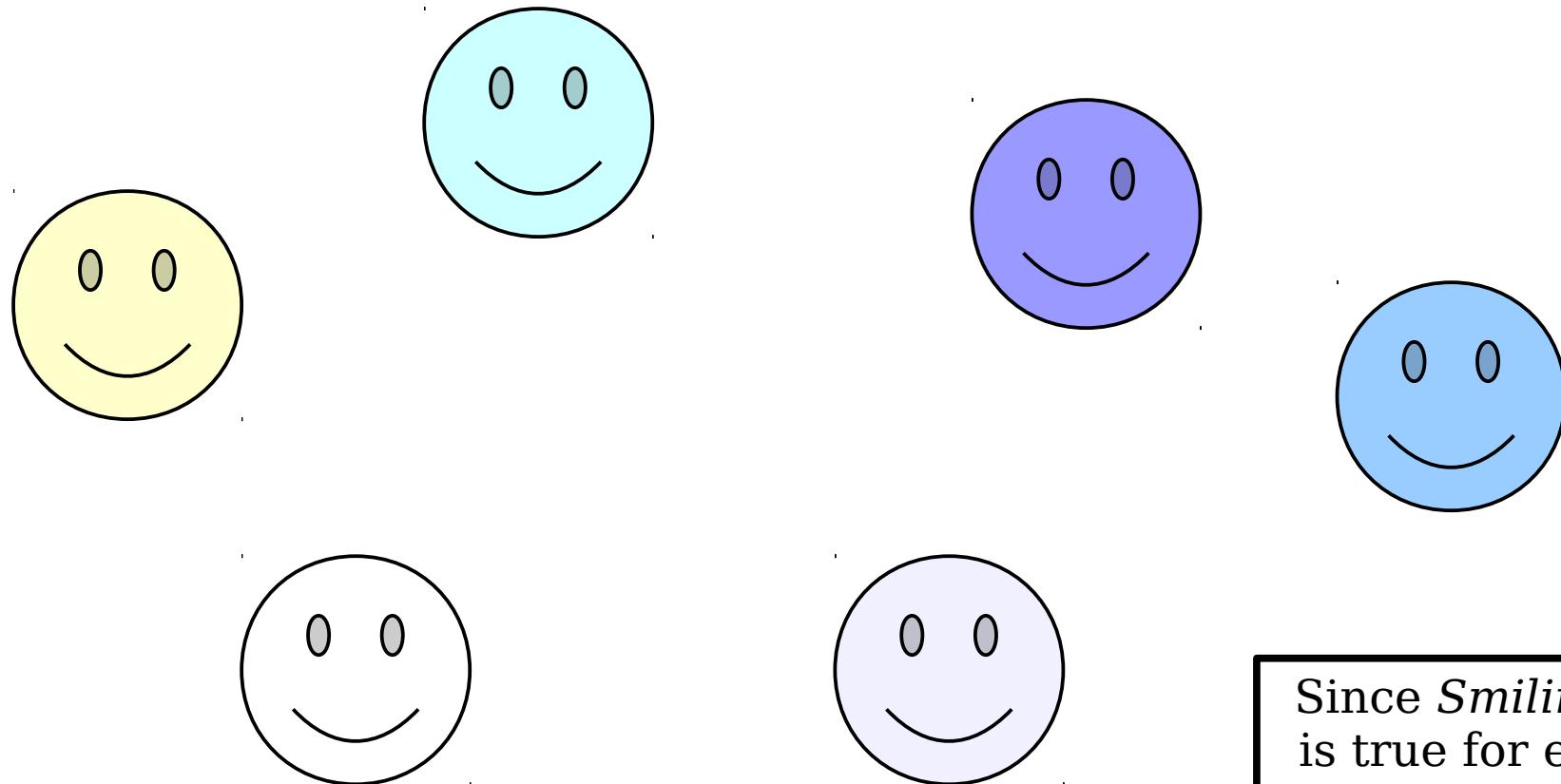
The Universal Quantifier



The Universal Quantifier

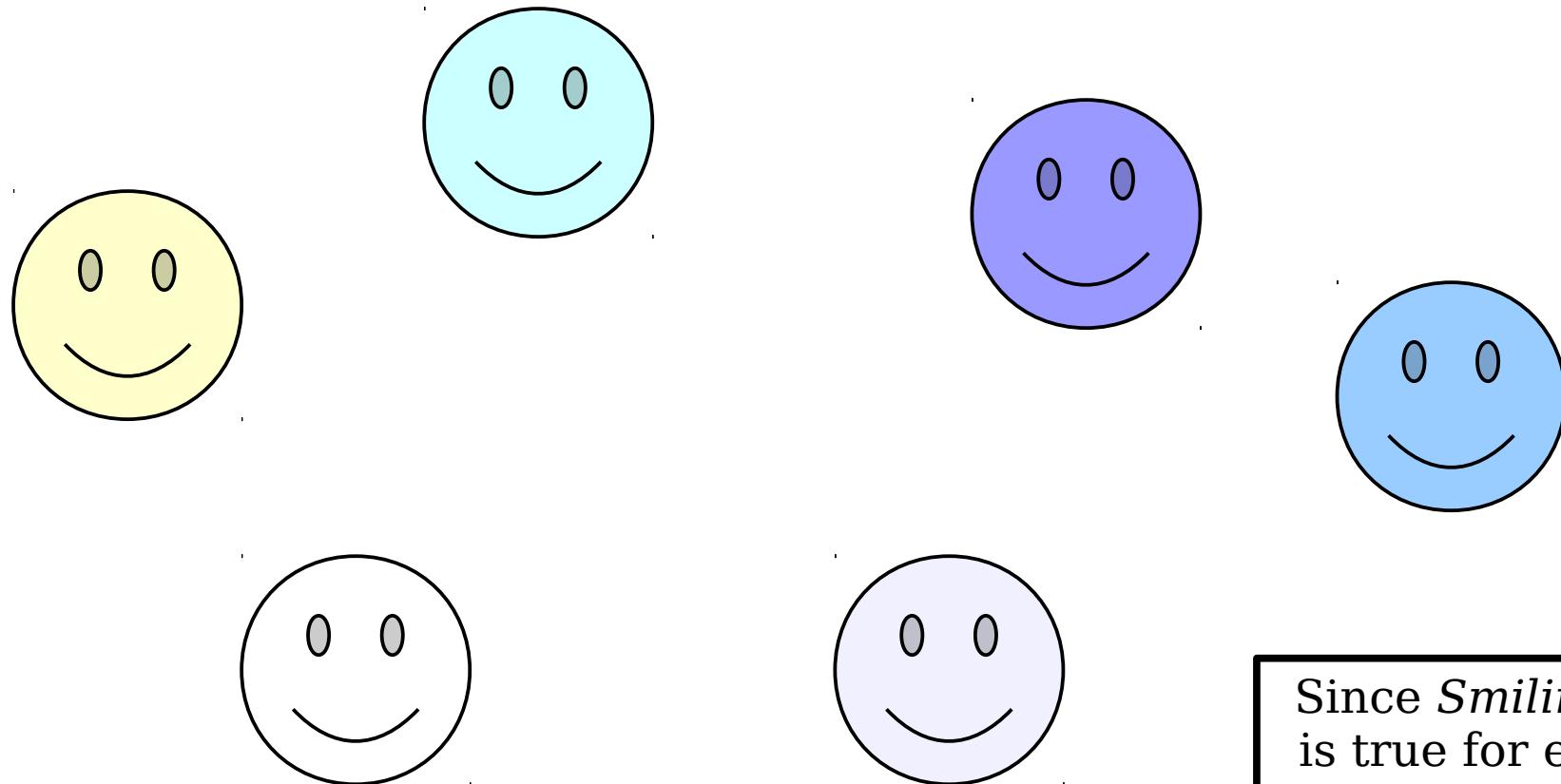


The Universal Quantifier


$$\forall x. \text{Smiling}(x)$$

Since $\text{Smiling}(x)$ is true for every choice of x , this statement evaluates to true.

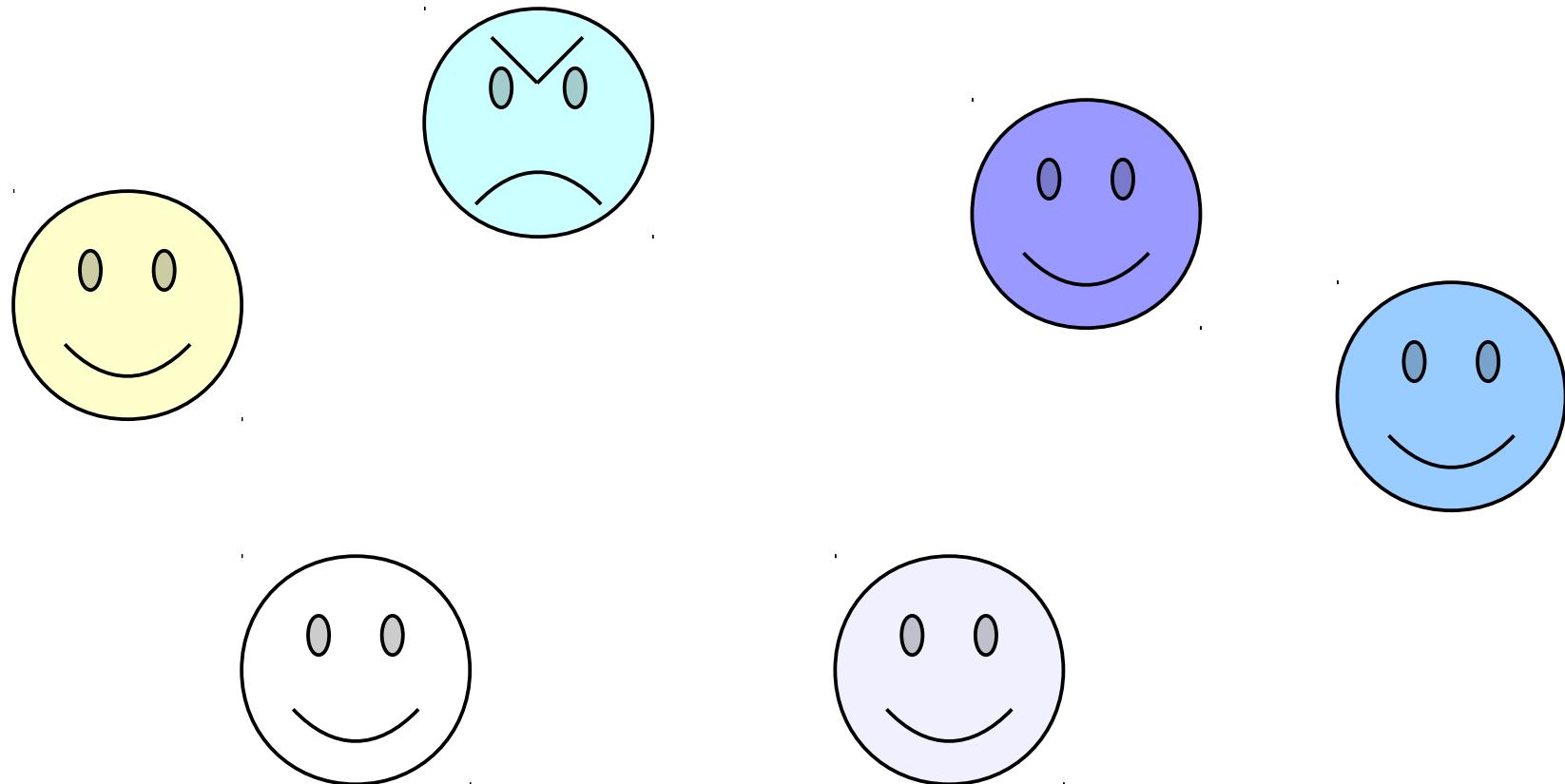
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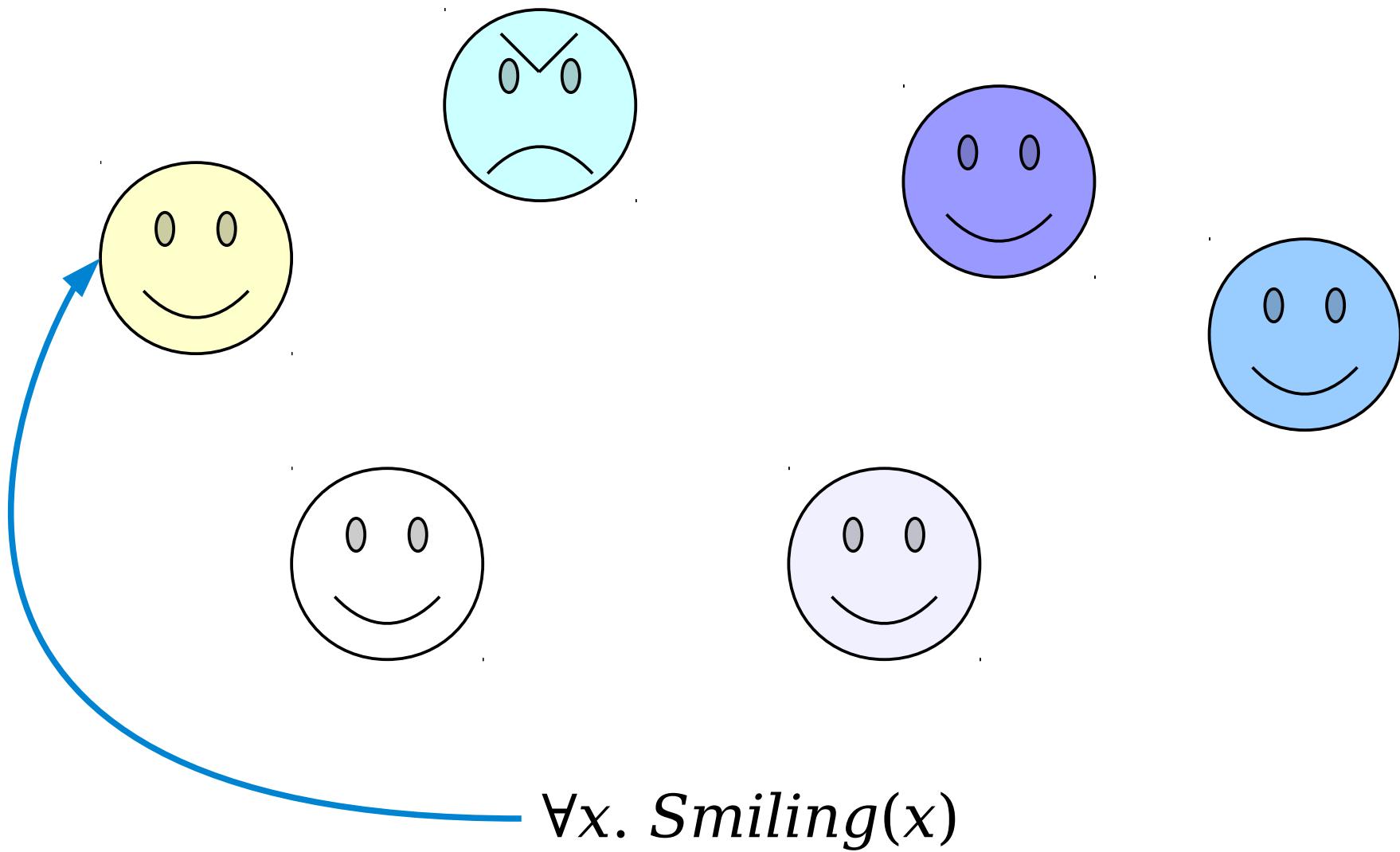
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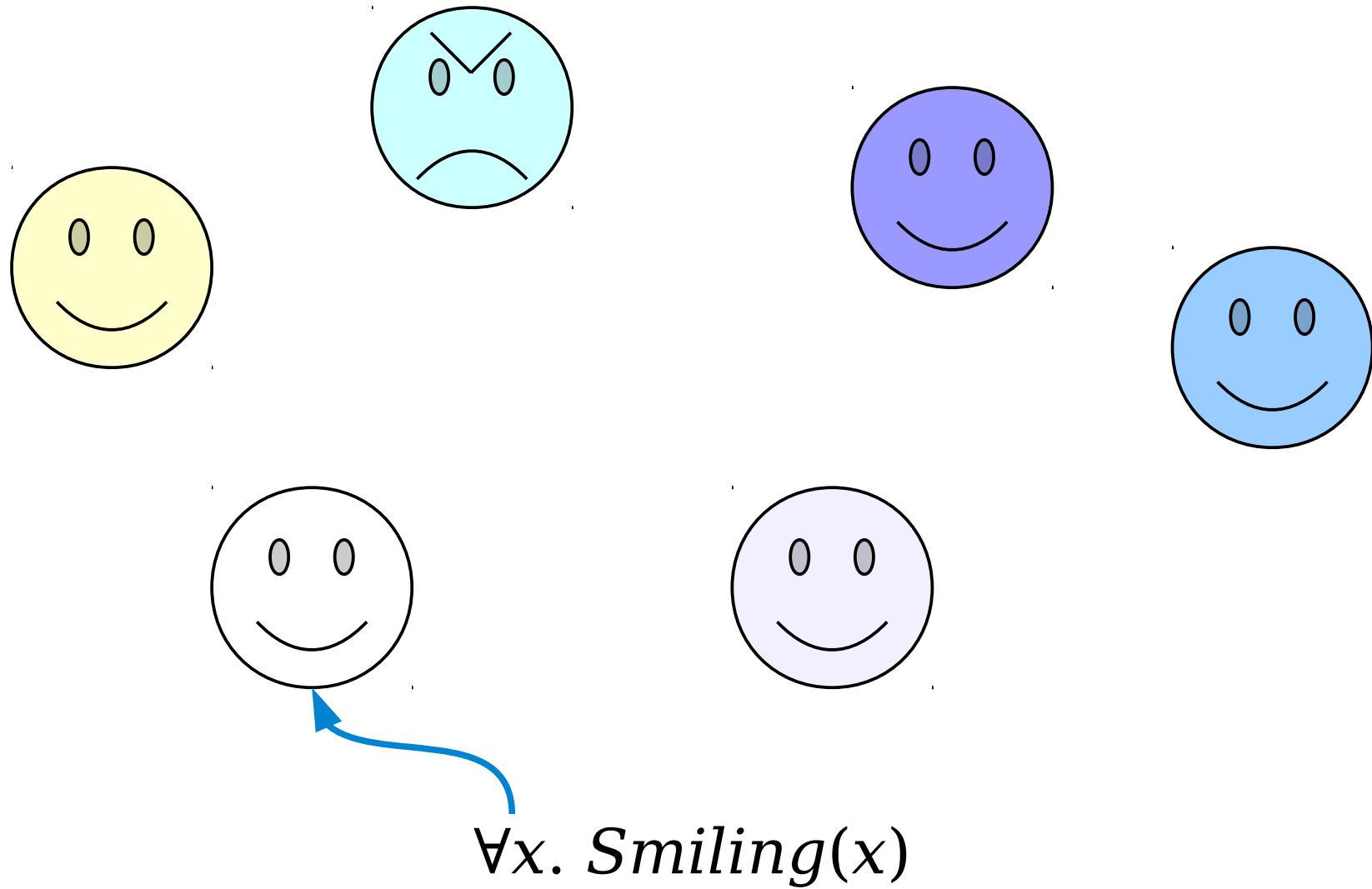
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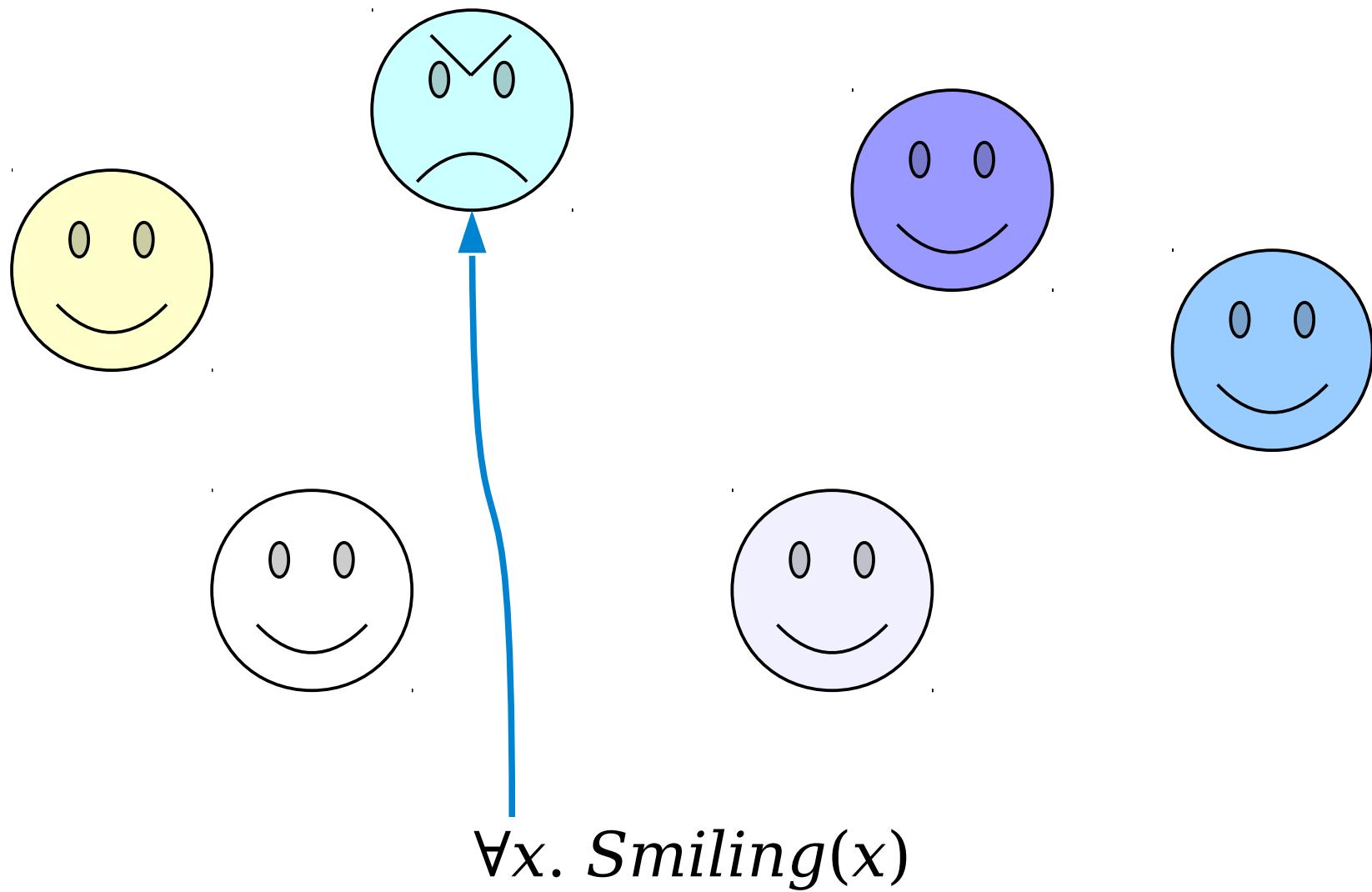
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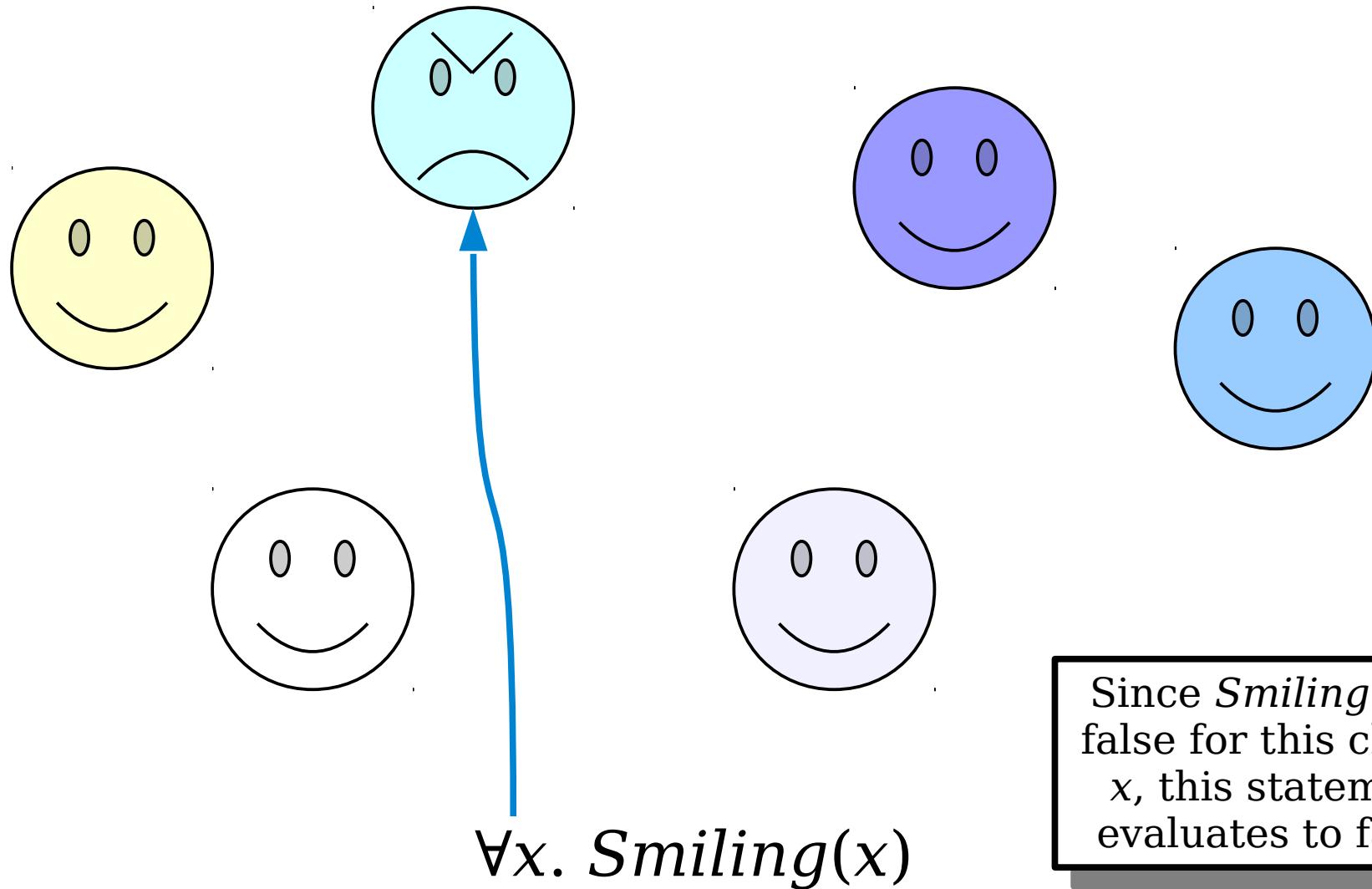
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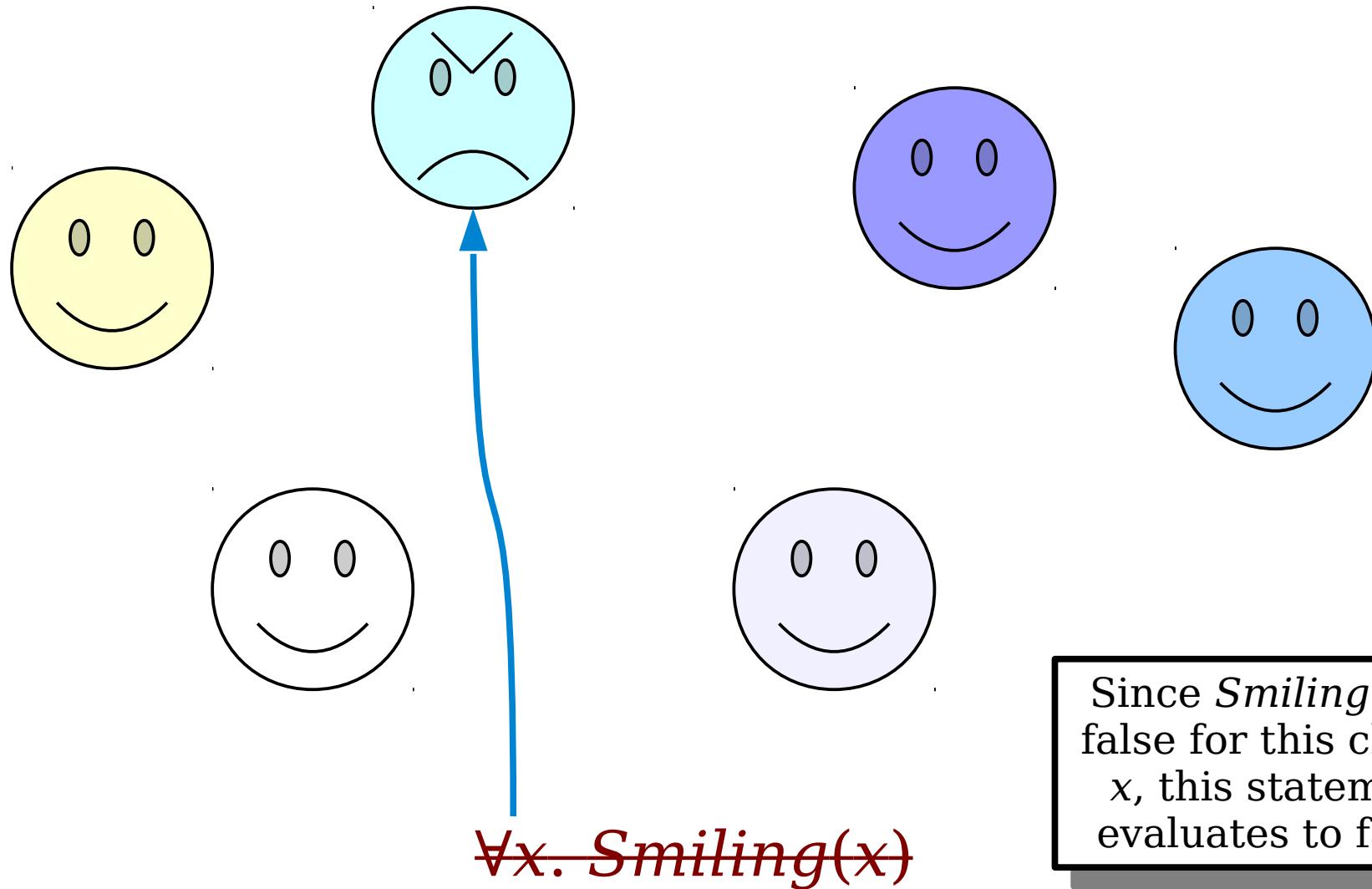
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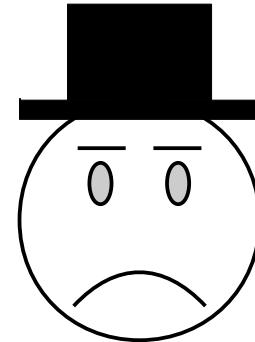
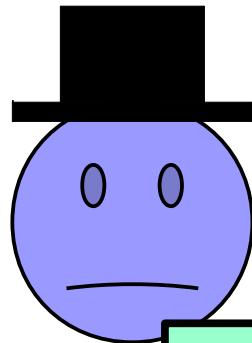
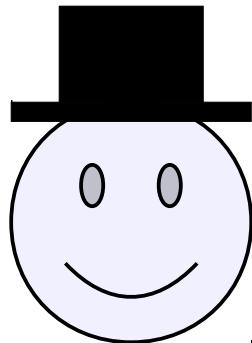
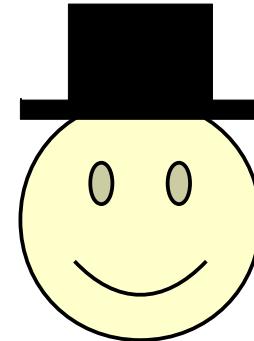
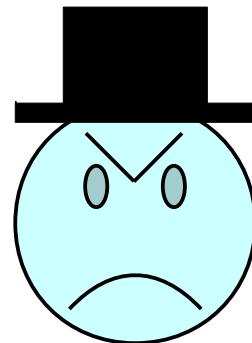
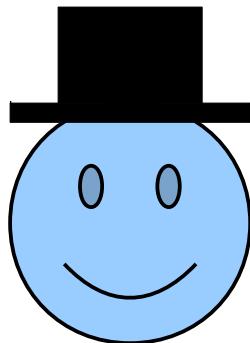
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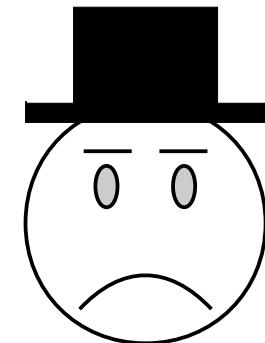
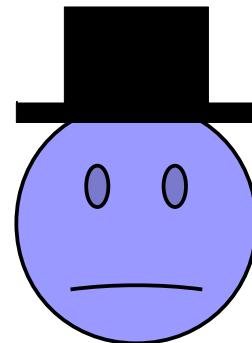
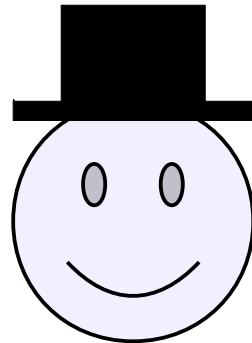
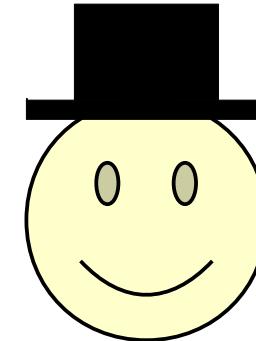
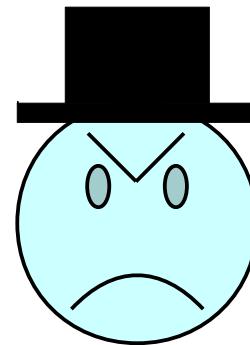
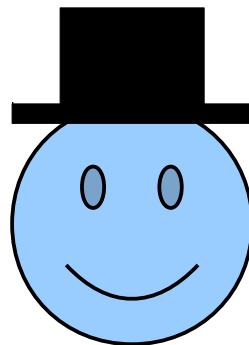
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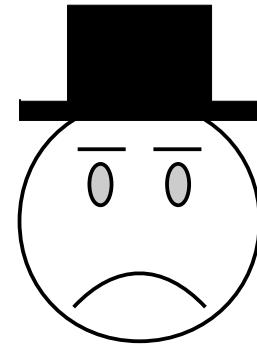
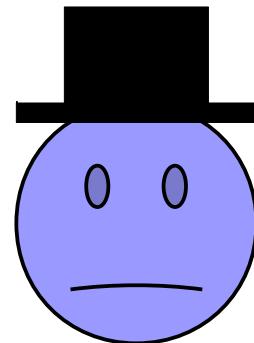
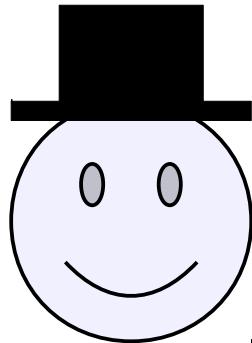
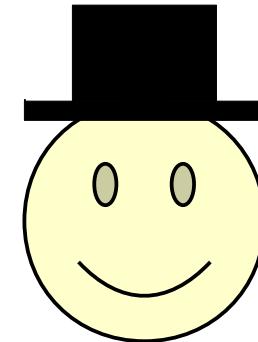
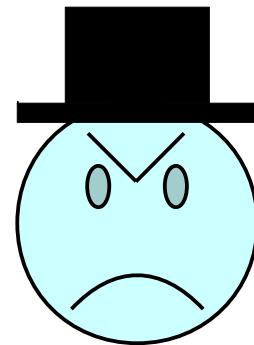
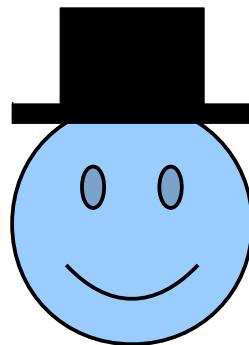
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$(\forall x. Smiling(x)) \rightarrow (\forall y. WearingHat(y))$

The Universal Quantifier


$$(\forall x. Smiling(x)) \rightarrow (\forall y. WearingHat(y))$$

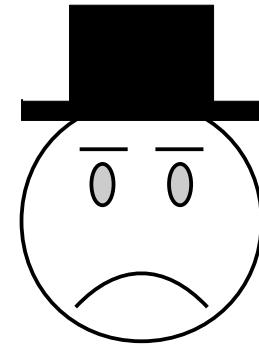
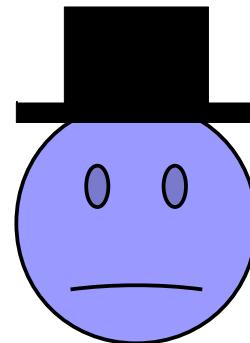
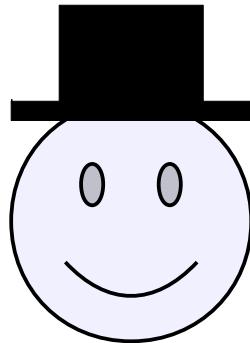
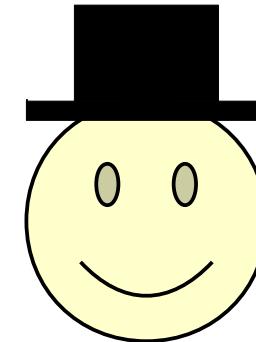
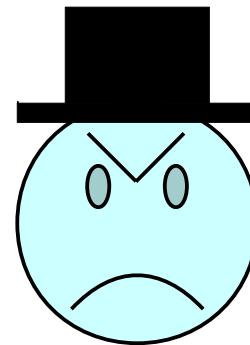
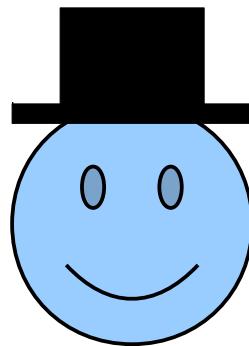
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Is this part of the statement true or false?

$(\forall x. Smiling(x)) \rightarrow (\forall y. WearingHat(y))$

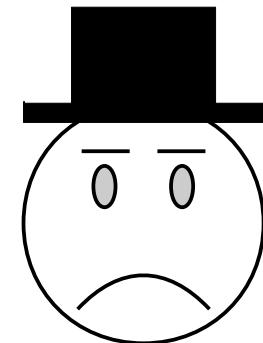
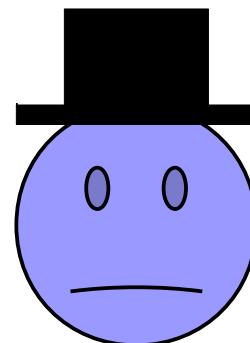
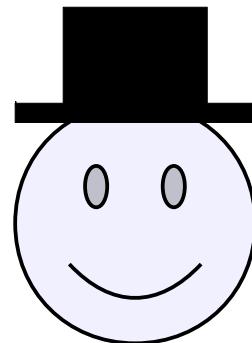
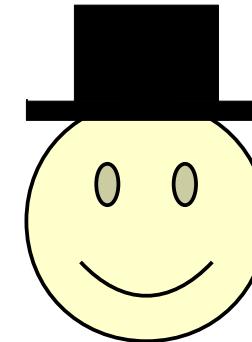
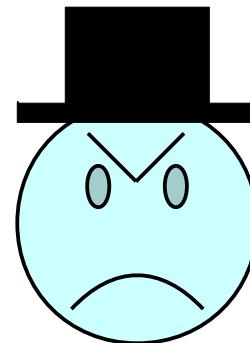
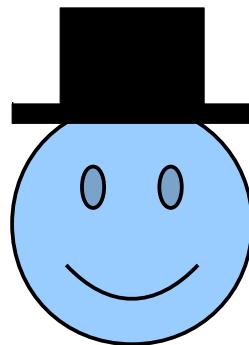
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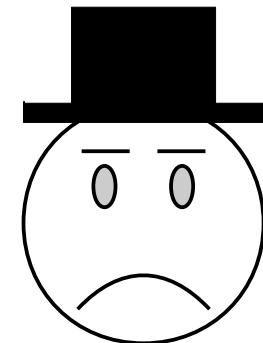
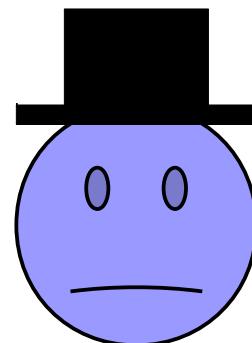
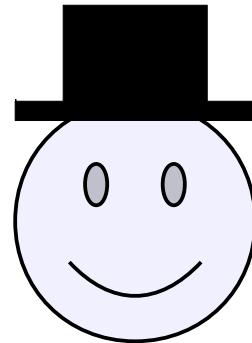
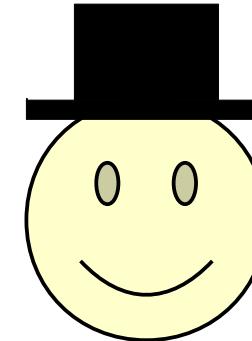
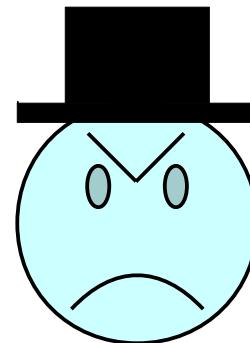
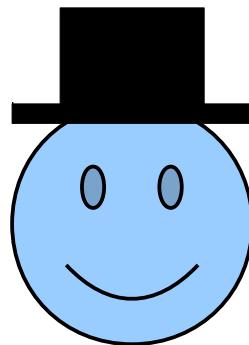
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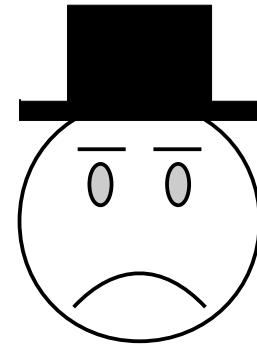
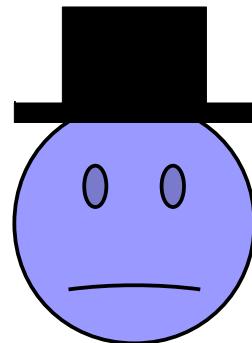
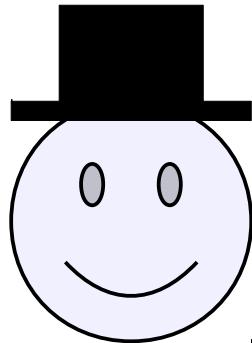
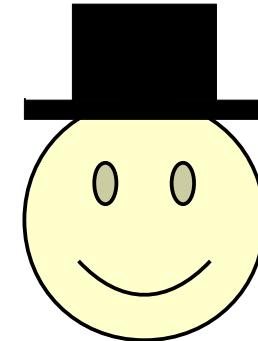
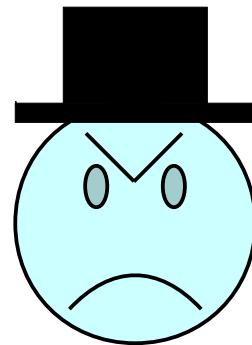
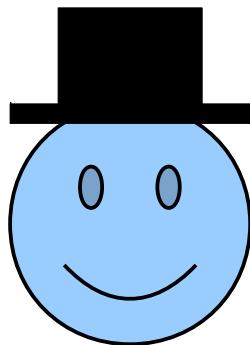
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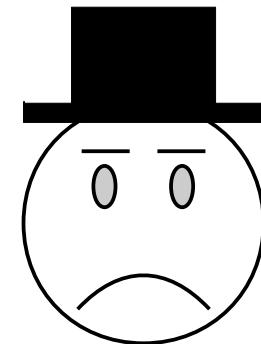
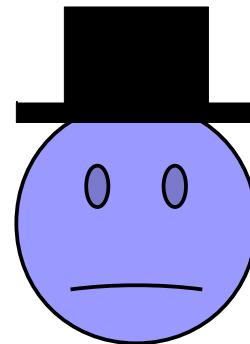
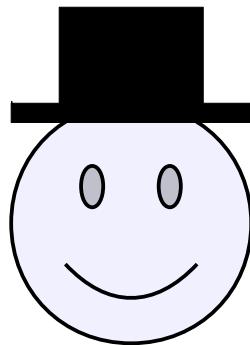
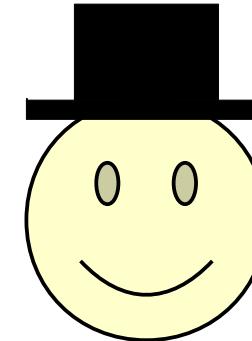
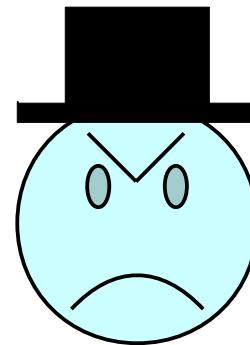
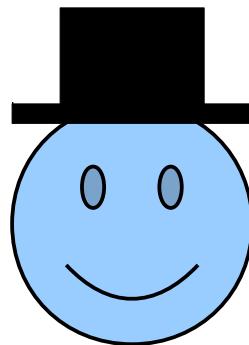
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Is this overall statement true or false in this scenario?

$(\forall x. Smiling(x)) \rightarrow (\forall y. WearingHat(y))$

The Universal Quantifier



Is this overall statement true or false in this scenario?

$(\forall x. Smiling(x)) \rightarrow (\forall y. WearingHat(y))$

Fun with Edge Cases

$\forall x. Smiling(x)$

Fun with Edge Cases

Universally-quantified statements are said to be ***vacuously true*** in empty worlds.

$\forall x. Smiling(x)$

Translating into First-Order Logic

Translating Into Logic

- First-order logic is an excellent tool for manipulating definitions and theorems to learn more about them.
- Need to take a negation? Translate your statement into FOL, negate it, then translate it back.
- Want to prove something by contrapositive? Translate your implication into FOL, take the contrapositive, then translate it back.

Translating Into Logic

- When translating from English into first-order logic, we recommend that you
think of first-order logic as a mathematical programming language.
- Your goal is to learn how to combine basic concepts (quantifiers, connectives, etc.) together in ways that say what you mean.

Using the predicates

- $\text{Smiling}(x)$, which states that x is smiling, and
- $\text{WearingHat}(x)$, which states that x is wearing a hat,

write a sentence in first-order logic that says

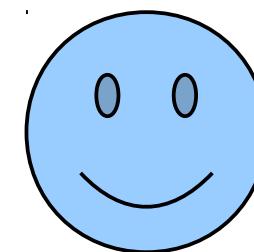
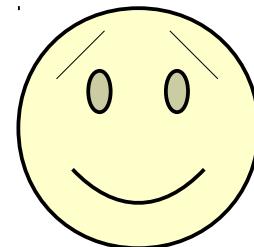
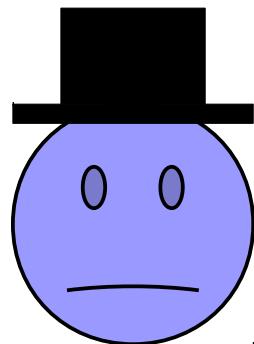
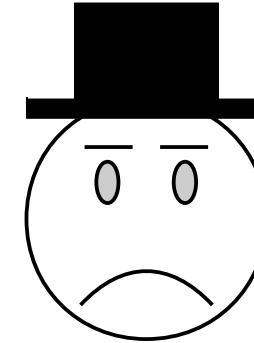
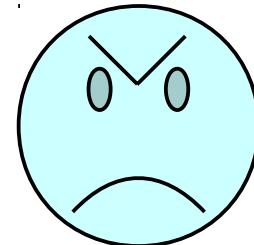
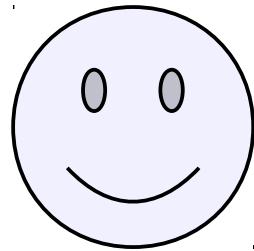
some smiling person wears a hat.

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“Some smiling person wears a hat.”

$\exists x. (Smiling(x) \wedge WearingHat(x))$

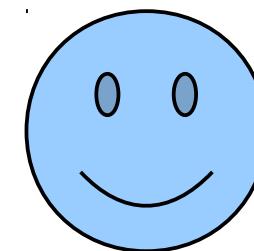
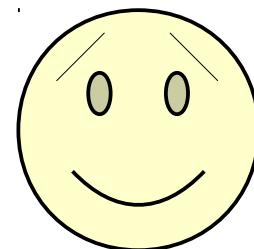
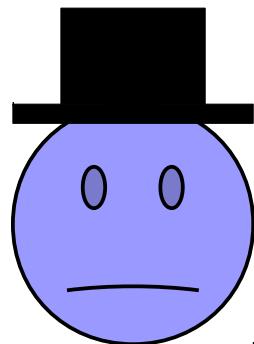
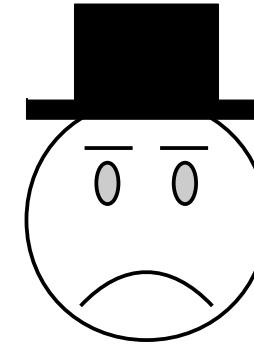
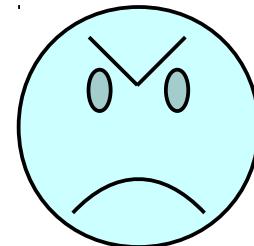
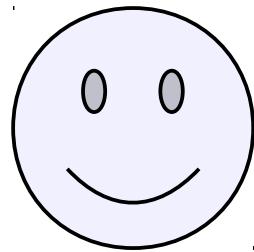
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“Some smiling person wears a hat.”

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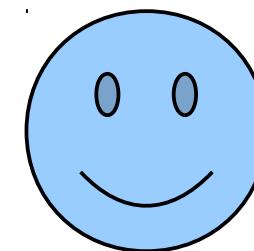
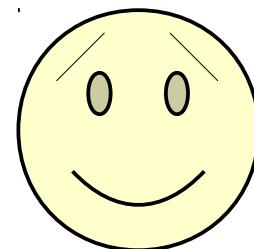
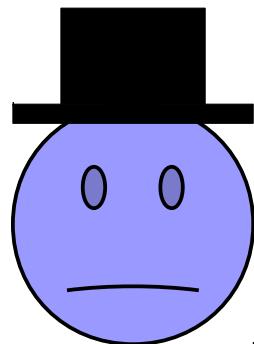
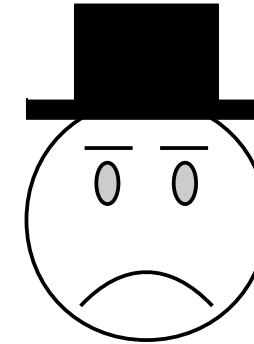
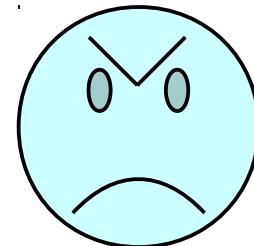
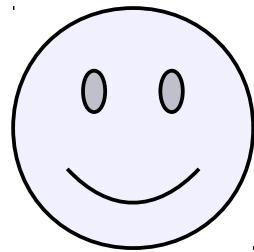
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“Some smiling person wears a hat.”

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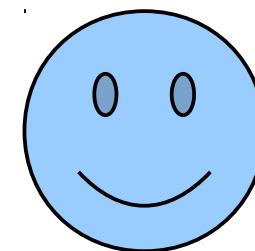
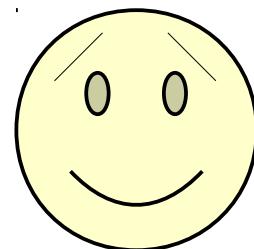
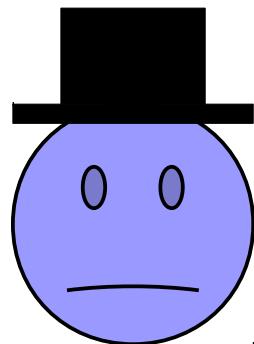
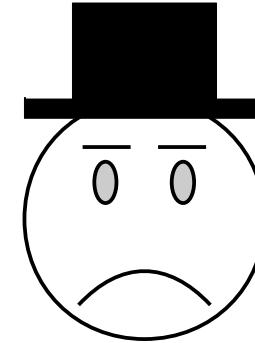
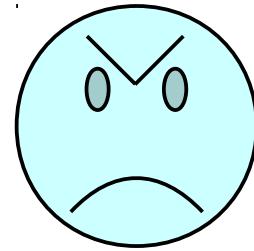
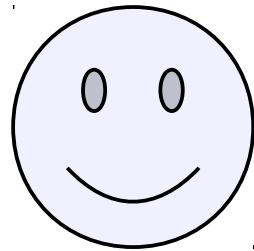
$$\exists x. (Smiling(x) \rightarrow WearingHat(x))$$



“Some smiling person wears a hat.” ***False***

$$\exists x. (Smiling(x) \wedge WearingHat(x))$$

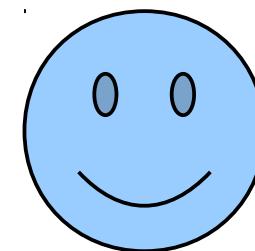
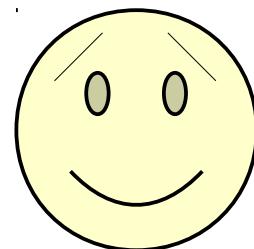
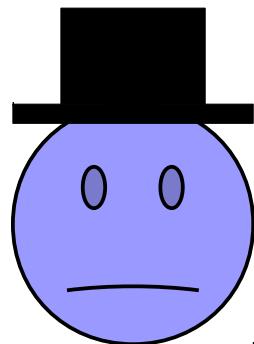
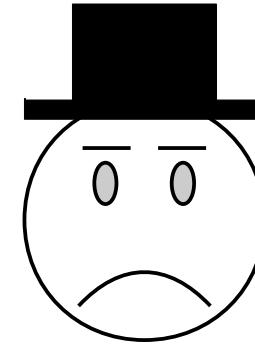
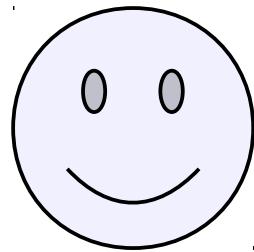
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“Some smiling person wears a hat.” ***False***

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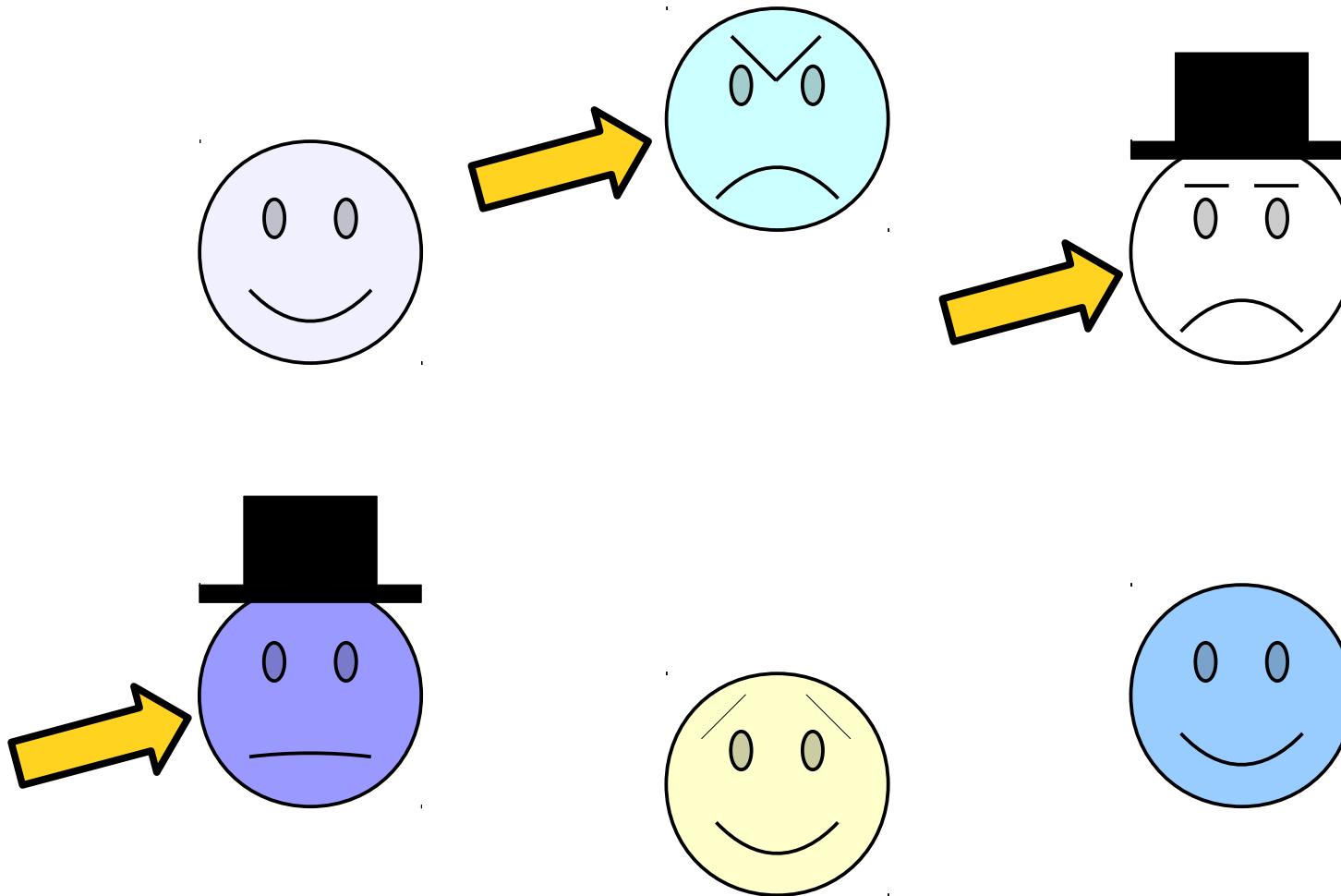
$\exists x. (Smiling(x) \rightarrow WearingHat(x))$



“Some smiling person wears a hat.” ***False***

$\exists x. (Smiling(x) \wedge WearingHat(x))$ ***False***

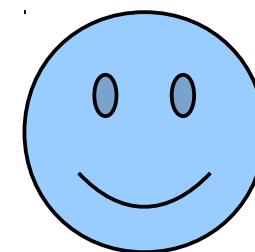
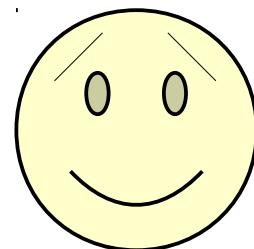
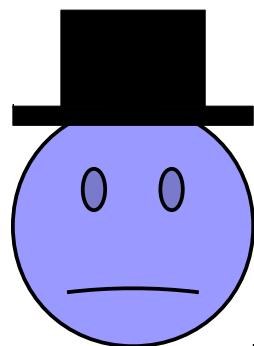
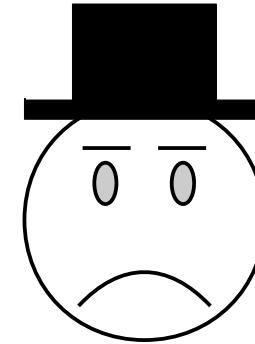
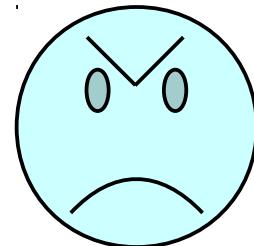
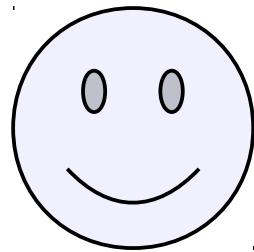
$\exists x. (Smiling(x) \rightarrow WearingHat(x))$



“Some smiling person wears a hat.” ***False***

$\exists x. (Smiling(x) \wedge WearingHat(x))$ ***False***

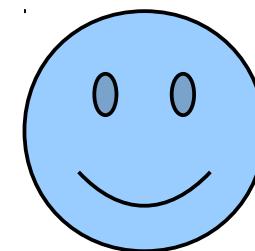
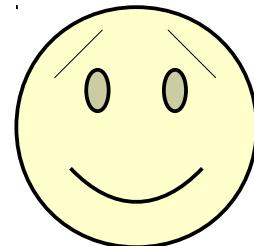
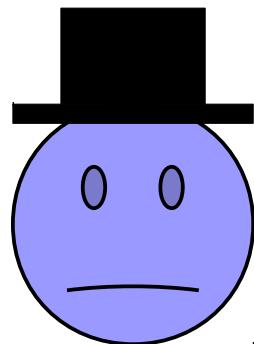
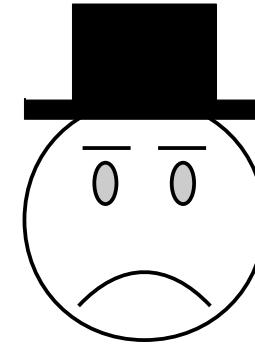
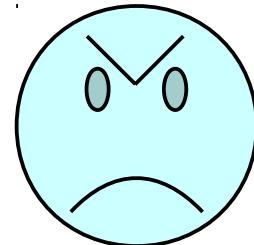
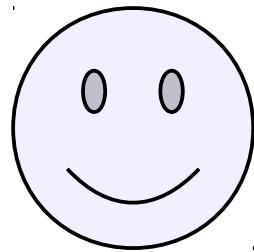
$\exists x. (Smiling(x) \rightarrow WearingHat(x))$ ***True***



“Some smiling person wears a hat.” ***False***

$\exists x. (Smiling(x) \wedge WearingHat(x))$ ***False***

$\exists x. (Smiling(x) \rightarrow WearingHat(x))$ ***True***



“Some smiling person wears a hat.” ***False***

$\exists x. (Smiling(x) \wedge WearingHat(x))$ ***False***

~~$\exists x. (Smiling(x) \rightarrow WearingHat(x))$~~ ***True***

“Some P is a Q ”

translates as

$\exists x. (P(x) \wedge Q(x))$

Useful Intuition:

Existentially-quantified statements are false unless there's a positive example.

$$\exists x. (P(x) \wedge Q(x))$$

If x is an example, it must have property P on top of property Q .

Using the predicates

- $Smiling(x)$, which states that x is smiling, and
- $WearingHat(x)$, which states that x is wearing a hat,

write a sentence in first-order logic that says

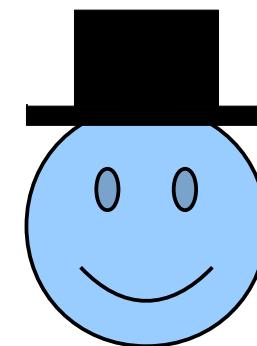
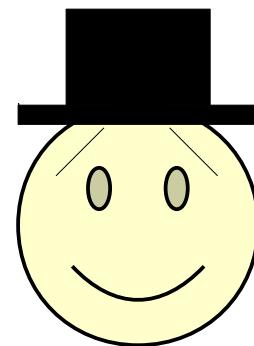
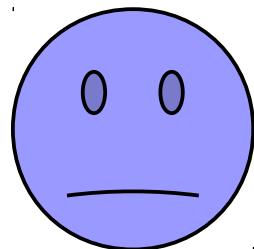
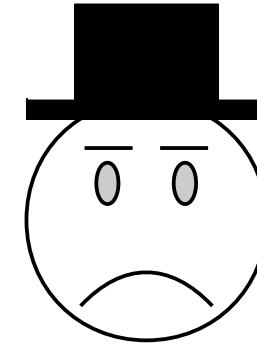
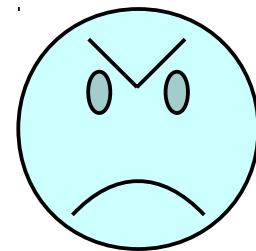
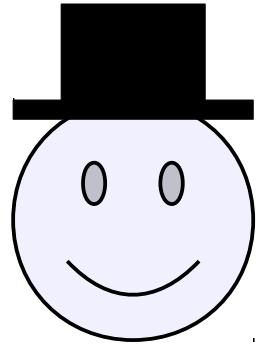
every smiling person wears a hat.

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“Every smiling person wears a hat.”

$\forall x. (Smiling(x) \wedge WearingHat(x))$

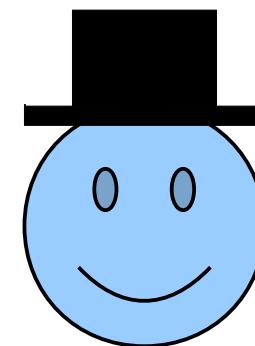
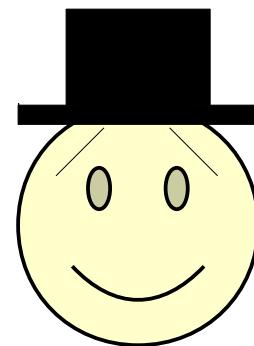
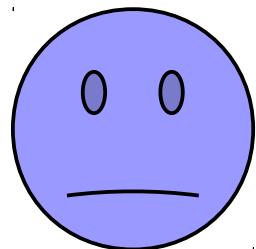
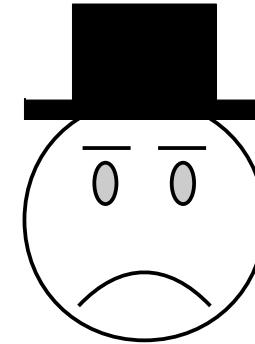
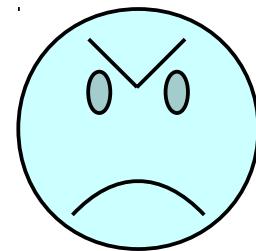
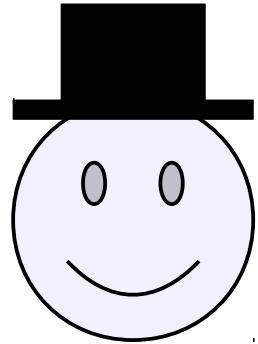
$\forall x. (Smiling(x) \rightarrow WearingHat(x))$



“Every smiling person wears a hat.”

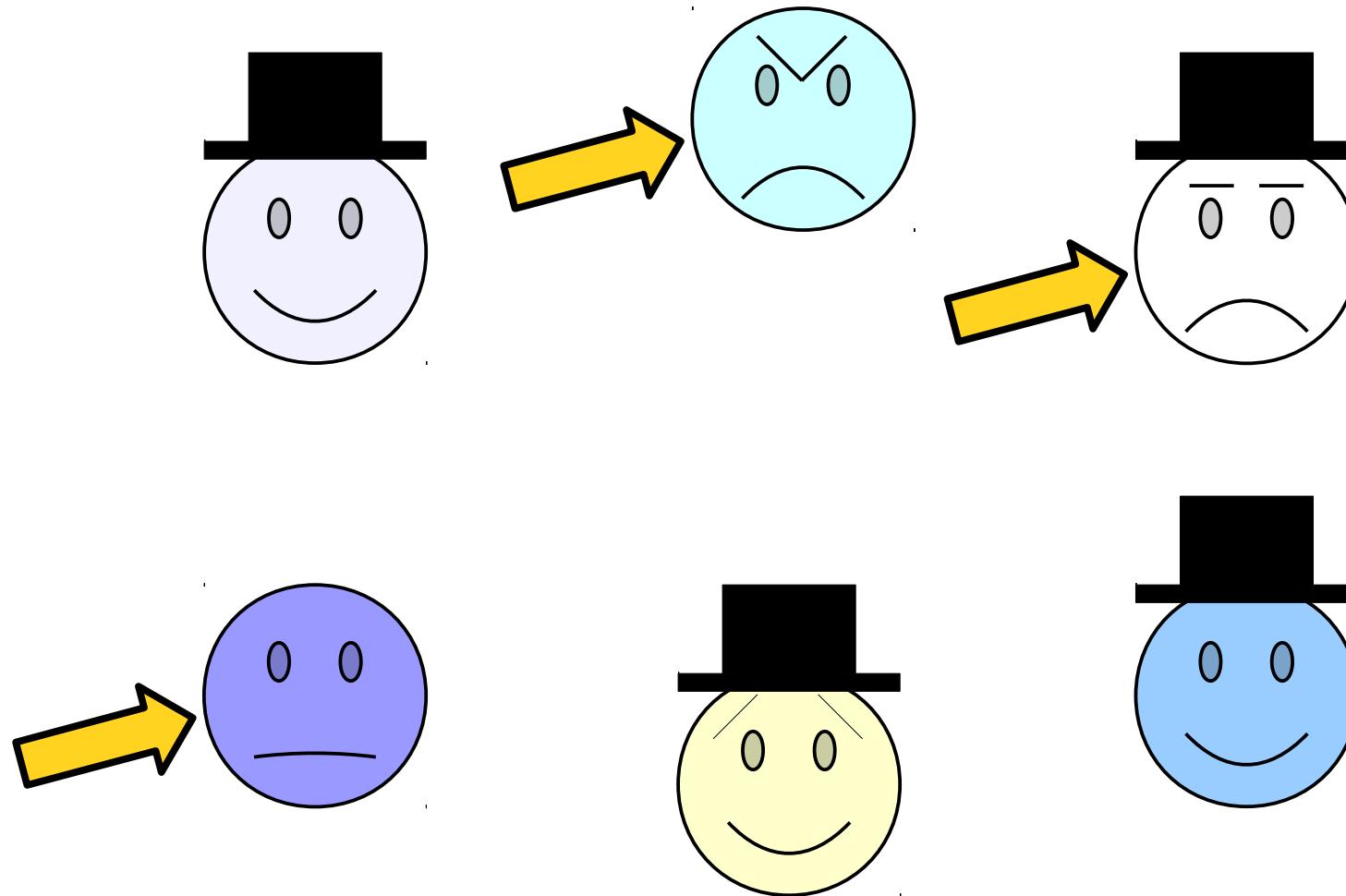
$$\forall x. (Smiling(x) \wedge WearingHat(x))$$

$$\forall x. (Smiling(x) \rightarrow WearingHat(x))$$



“Every smiling person wears a hat.” **True**

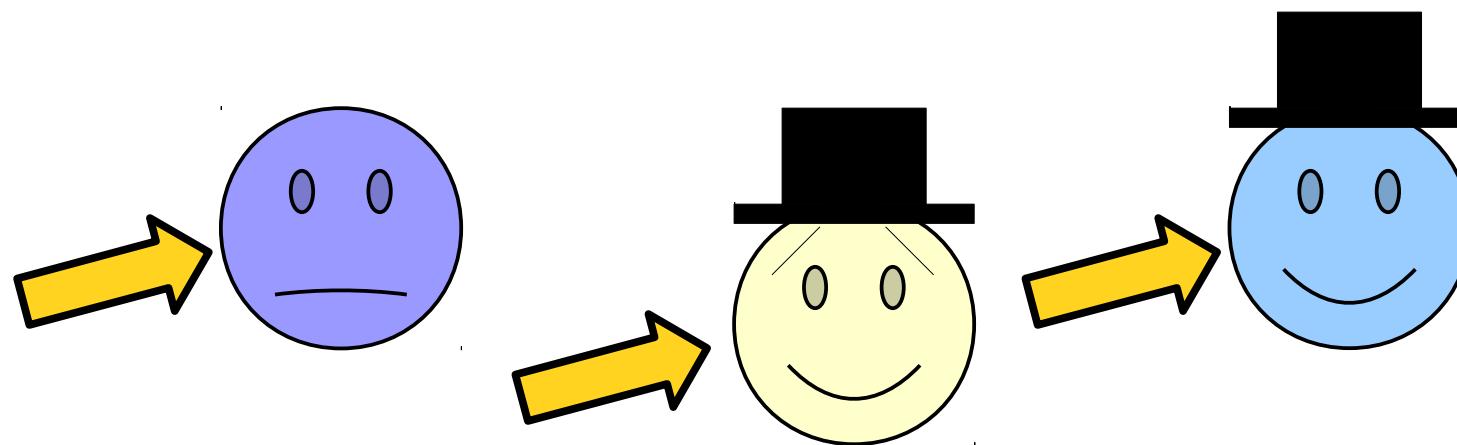
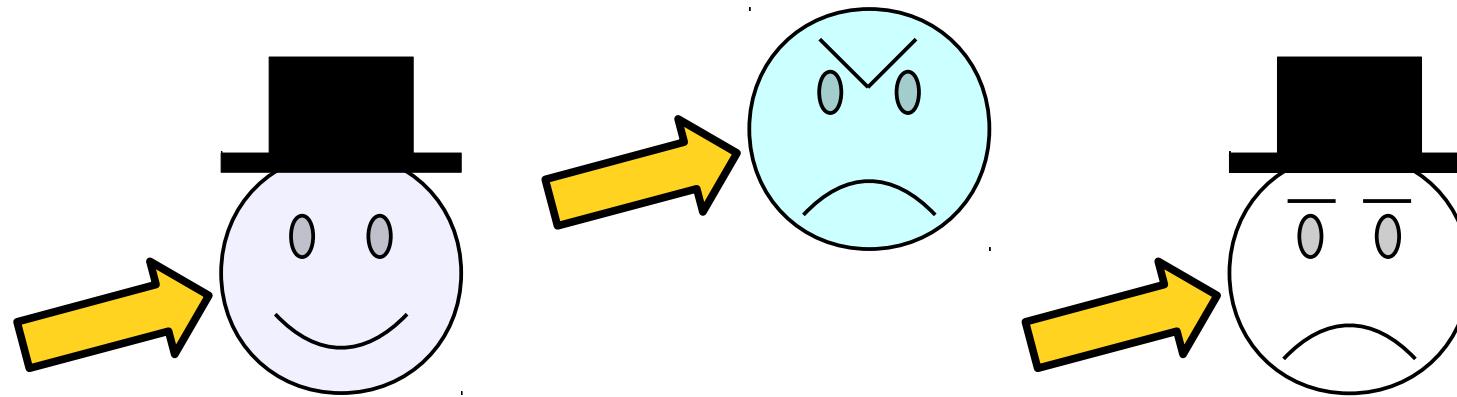
$$\forall x. (Smiling(x) \wedge WearingHat(x))$$
$$\forall x. (Smiling(x) \rightarrow WearingHat(x))$$



“Every smiling person wears a hat.” **True**

$\forall x. (Smiling(x) \wedge WearingHat(x))$ **False**

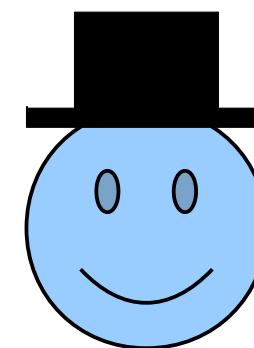
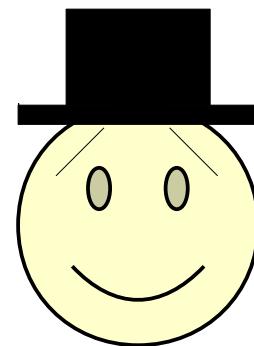
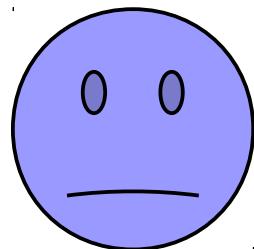
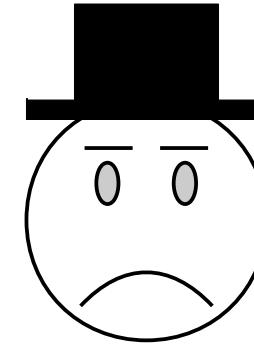
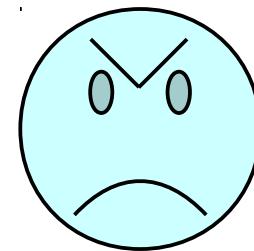
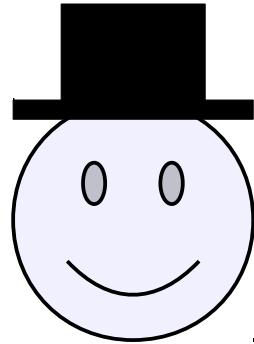
$\forall x. (Smiling(x) \rightarrow WearingHat(x))$



“Every smiling person wears a hat.” **True**

$\forall x. (Smiling(x) \wedge WearingHat(x))$ **False**

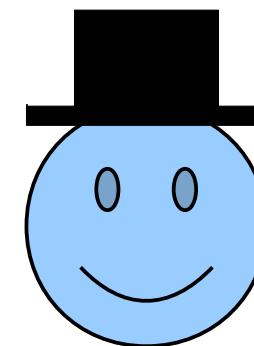
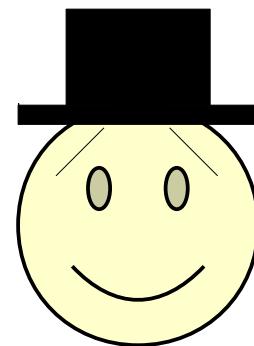
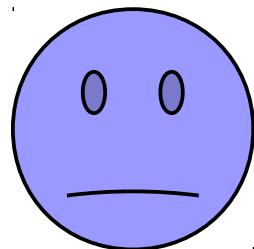
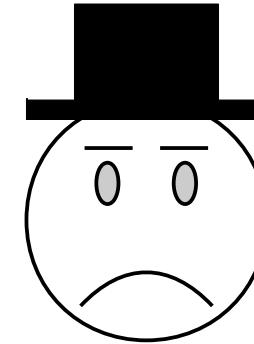
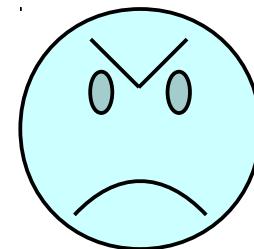
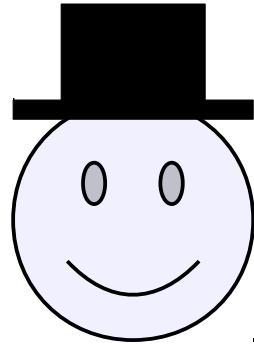
$\forall x. (Smiling(x) \rightarrow WearingHat(x))$ **True**



“Every smiling person wears a hat.” **True**

$\forall x. (Smiling(x) \wedge WearingHat(x))$ **False**

$\forall x. (Smiling(x) \rightarrow WearingHat(x))$ **True**



“Every smiling person wears a hat.” **True**

~~$\forall x. (Smiling(x) \wedge WearingHat(x))$~~ **False**

$\forall x. (Smiling(x) \rightarrow WearingHat(x))$ **True**

“All P 's are Q 's”

translates as

$\forall x. (P(x) \rightarrow Q(x))$

Useful Intuition:

Universally-quantified statements are true unless there's a counterexample.

$$\forall x. (P(x) \rightarrow Q(x))$$

If x is a counterexample, it must have property P but not have property Q .

Good Pairings

- The \forall quantifier *usually* is paired with \rightarrow .

$$\forall x. (P(x) \rightarrow Q(x))$$

- The \exists quantifier *usually* is paired with \wedge .

$$\exists x. (P(x) \wedge Q(x))$$

- In the case of \forall , the \rightarrow connective prevents the statement from being *false* when speaking about some object you don't care about.
- In the case of \exists , the \wedge connective prevents the statement from being *true* when speaking about some object you don't care about.

Next Time

- ***First-Order Translations***
 - How do we translate from English into first-order logic?
- ***Quantifier Orderings***
 - How do you select the order of quantifiers in first-order logic formulas?
- ***Negating Formulas***
 - How do you mechanically determine the negation of a first-order formula?
- ***Expressing Uniqueness***
 - How do we say there's just one object of a certain type?